JET PROPULSION

Journal of the

—AMERICAN ROCKET SOCIETY—

Rocketry . . . . . Jet Propulsion Sciences . . . . . Astronautics

Hypersonic Aerodynamics Issue

Gas Dynamics Laboratory, Princeton University
Hypersonic flow at Mach 12.7
Helium wind tunnel at 1000 psi stagnation pressure

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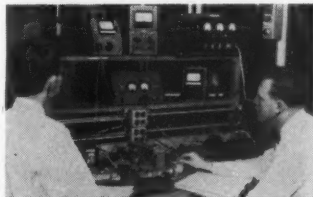
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Hypersonic Studies of the Leading Edge Effect on the Flow Over a Flat Plate

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Forrestal Research Center, Princeton University, Princeton, N. J.

Using the Princeton helium hypersonic wind tunnel, the effect of leading edge thickness on the flow over a flat plate has been investigated. The study was carried out by optical measurements of the shock position over a wide range of leading edge thicknesses and surface pressure measurements for the thicker leading edges. The flow over the plate with a thick leading edge was found to be essentially inviscid, but viscosity was important for the case of thin leading edges. The strong inviscid effect caused by the finite thickness of the leading edge seemed to be sufficient to explain the discrepancies between viscous theory and experiment in the region near the leading edge.

Nomenclature

C = linear viscosity coefficient in relation $\mu_w/\mu_1 = C(T_w/T_1)$
d_i = boundary layer parameter depending on surface temperature
M = Mach number
p = pressure
Δp = static pressure increment
R_t = Reynolds number
t = leading edge thickness, inches
T = temperature
x = distance from leading edge
y = distance of shock above plate surface
u = velocity divided by maximum velocity on expanding to zero pressure
γ = specific heat ratio
δ^* = boundary layer displacement thickness
μ = viscosity

Subscripts

1 = tunnel empty condition
o = stagnation
t = based on leading edge thickness
w = condition on wall
x = based on distance from leading edge

Introduction

THE analysis of flows at many times the speed of sound are complicated by special problems which are unimportant at lower speeds. These problems can be broken down into two classes: (a) fluid dynamics problems, which depend on the conventional parameters of Mach number, Reynolds number, Nusselt number, and specific heat ratio; (b) molecular problems, which depend on temperature and involve the change of molecular structure at high temperatures.

The hypersonic fluid dynamic problems are governed by the same relations as other fluid dynamic problems. Considerable study has been given to the predominant fluid dynamics problems in the subsonic and supersonic range. At hypersonic speeds the relative magnitude of the various effects may be considerably different. This change in emphasis leads to several new problems which have received little attention. Effects of little importance at supersonic speeds may become dominant at hypersonic speeds, and vice versa.

The molecular problems are essentially new in aerodynamic applications. These problems only occur at high temperatures where the molecular structure may be considerably different than at conventional temperatures. Some of these problems have been encountered at the relatively low speeds and high temperatures which exist in rocket motors.

This paper describes a study of the flow about the leading edge of a flat plate and the experimental equipment used for this study. This flow involves several hypersonic fluid dynamics problems. At hypersonic speeds most shocks are strong and the entropy changes through the shocks cannot be neglected. The reflected waves, caused by the interaction of other waves with the strong shocks, are also important. Viscous forces are large and can cause important modifications in the inviscid external flow. All these factors combine in the flow about the leading edge of a flat plate. The study of this problem was carried out in a hypersonic wind tunnel using helium as the working fluid. The tunnel operates in a Reynolds number range where the fluid is essentially a continuum and slip phenomena are limited to very small distances. The work reported herein covers an analysis of shock shapes and pressure distributions of the hypersonic flow about a flat plate over a wide range of Reynolds numbers and leading edge thicknesses.

Hypersonic Wind Tunnel

General Discussion

There are several techniques for making fluid dynamics experiments which have been proved useful. Wind tunnels, shock tubes, and free flight experiments all have important uses. The wind tunnel has proved to be very valuable for many investigations in the subsonic and supersonic range and

might be expected to be equally useful in the hypersonic range. The wind tunnel allows tests to be made over relatively long periods, under controlled conditions, with ease and nominal expense.

The main problem in operating a wind tunnel hypersonically is to heat the air enough so that it does not condense after expanding to high velocities. Stagnation temperatures of the order of 1300 to 1500 F are required to prevent condensation of the air in the test section at Mach number 10. To obtain stagnation temperatures high enough to study the molecular problems of hypersonic flow, of the order of 4000 to 6000 F, seems to be impractical in a wind tunnel. For this reason hypersonic wind tunnels are limited to a study of only the fluid dynamic problems. Another technique for avoiding condensation in the working section of a wind tunnel is to use a gas which does not condense. Since the wind tunnel cannot simulate the full flight problem, there is little loss for the purpose of fluid dynamics research in using a gas other than air. If a gas with low condensation temperature such as helium is used, Mach numbers in excess of 20 can be obtained at atmospheric stagnation temperatures. The elimination of the heater and the accompanying problems of a hot tunnel is such an important simplification that the helium tunnel seems to be the most satisfactory device for fluid dynamics research at Mach numbers greater than 10. The results obtained in the helium tunnel do not represent the complete answer to hypersonic flight but only lead to an understanding of the fluid dynamics problems. The data obtained from such a tunnel are directly comparable to the data obtained in hot air tunnels at the same Mach, Reynolds, and Nusselt numbers, corrected for the effect of the different specific heat ratio.

Princeton Helium Hypersonic Tunnel

The Princeton hypersonic wind tunnel is of the blow-down type using helium as the test fluid. Fluid dynamics problems may be studied in this tunnel up to Mach 20. A detailed description of the tunnel is given in (15). The helium is obtained at high pressures in tank semitrailers which are used as the storage system. From the trailer, the helium is piped through a regulating valve to the settling chamber. The tunnel may be run at stagnation pressures up to 2000 psia for periods of about 10 minutes. A stagnation pressure range of 800 to 1400 psia has been used in these tests. To obtain the high pressure ratios which are needed, an air operated ejector, using air at 1500 psi from a second high pressure storage system, is used downstream of the hypersonic diffuser. A general layout of the piping and tunnel is shown in Fig. 1.
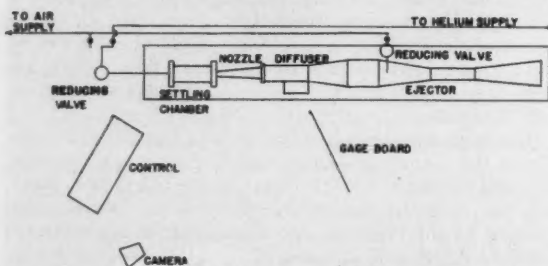
Fig. 1   Piping and control system for helium hypersonic tunnel

The first nozzle used, Fig. 2(a), was an axial symmetric conical nozzle with no windows and a maximum diameter of approximately 3 inches. The second nozzle, Fig. 2(b), is similar to the first one except that it has been cut open and windows installed. The disturbances from the windows do not effect the central core in the region of interest. The models and probe are supported in the test section by means of the sting and spider shown. The first nozzle was used on the initial surface pressure distribution tests and the second
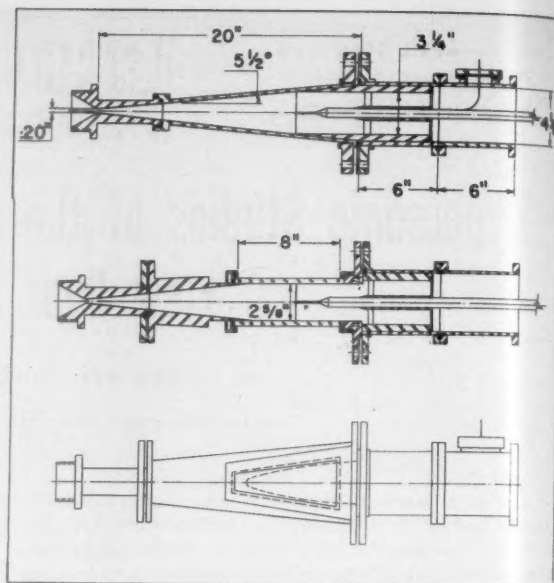
Fig. 2   Cross sections of hypersonic tunnel
Top: axial symmetric nozzle; *center and bottom:* axial symmetric nozzle with windows.
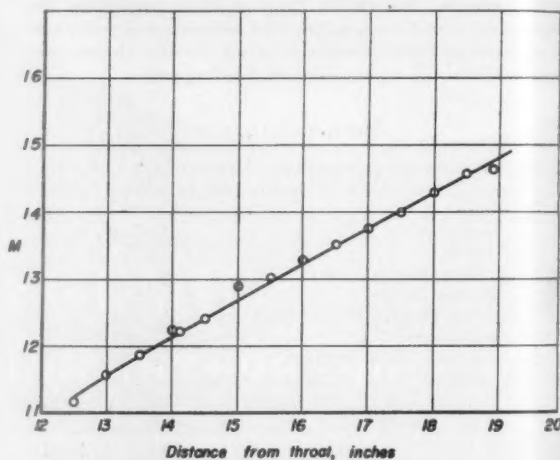
Fig. 3   Mach number distribution along the center line of the hypersonic tunnel

nozzle for the optical studies and additional pressure distribution measurements.

The Mach number along the center line of the nozzle is shown in Fig. 3. Since the nozzle was not contoured, a region of constant Mach number was not obtained and the tests are made in a region of Mach number gradient. The effects of this gradient will be discussed later. The diffuser, Fig. 2, is a straight tube with a central body sting support. This arrangement is probably not an optimum, but gave the best results of the limited varieties which were examined.

Experimental Study

A test program was undertaken to study the flow about the forepart of a flat plate at high Mach number. If only the adiabatic case is considered, there are three parameters which might influence the flow. These parameters are the Mach number, the Reynolds number, and the geometry. In the case of the semi-infinite flat plate, the only geometric parameter is the distance scale based on leading edge thickness.

The experimental model could only approach a semi-infinite plate since it had finite boundaries. In general, it would be expected that the effects of the tips and trailing edges would be limited to small regions and would not propagate over the entire plate. It would be expected that there should be considerable area which behaved as a semi-infinite plate. Again judging by lower speed data, the effect of the leading edge would be expected to be limited to a few leading edge thicknesses. Therefore, a sharp flat plate of reasonable aspect ratio might be expected to behave much like an infinitely sharp semi-infinite flat plate. (This is the model treated by several theoretical investigators (1) to (11)).

The experimental study undertaken was an analysis of the flow over a flat plate at variable leading edge thicknesses, Reynolds number, and Mach number. The effects of the tips and trailing edge were investigated in order to find a region which was free of these effects. The study was carried out by measurements of the shock shape and the surface pressure distributions. In order to study the effect of the leading edge thickness it was necessary to vary this thickness over a wide range. The leading edge thickness and stagnation pressures were varied over the widest ranges possible. The leading edge range was limited by the requirement that the leading edge had to be thick enough to remain unchanged during the run and could not be so thick that the tunnel was stalled. The stagnation pressure was varied between 1400 and 800 psia giving a Reynolds number per inch range of 1.15 to 0.65 million. To obtain sharp leading edges, a simple model with no pressure taps near the leading edge was used. This model could be hardened and honed on both surfaces. The model is shown in Fig. 4. Interferograms were taken of the model at Mach 12.7 at varying leading edge thicknesses and stagnation pressures. The leading edge thickness was varied from 0.17 to 59.5 thousandths of an inch. The blunting was accomplished by cutting off the leading edge perpendicular to the top surface. The measurement of the leading edge thickness was made with a microscope and calibrated eyepiece. A typical range of interferograms is presented in Fig. 5: "a" is for a thin leading edge, "b" a medium edge, and "c" for a thick leading edge. "a" shows a shock wave starting at or very close to the leading edge and a pronounced boundary layer indicated by the large fringe shift between the shock wave and surface. "c" shows a strong detached shock and almost no indication of a boundary layer. "b" is an intermediate position showing a blending together of the features found in "a" and "c." "a" shows the fringes approaching the plate perpendicularly. This does not mean that the flow is separated, but only that the density is so low at this point that the interferometer has not sufficient sensitivity to show the density changes in this region. A similar situation exists in "c." In this case, the density has been reduced to such a level outside of the boundary layer by the entropy changes through the strong shocks that the interferogram does not show the additional density changes through the boundary layer. These interferograms can be used only as qualitative indications of the density since the
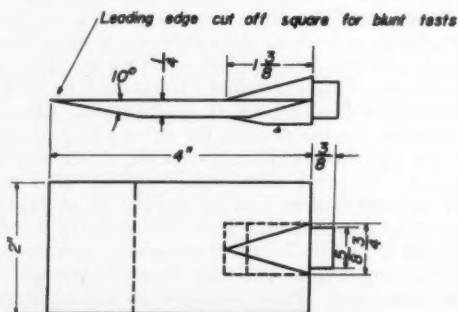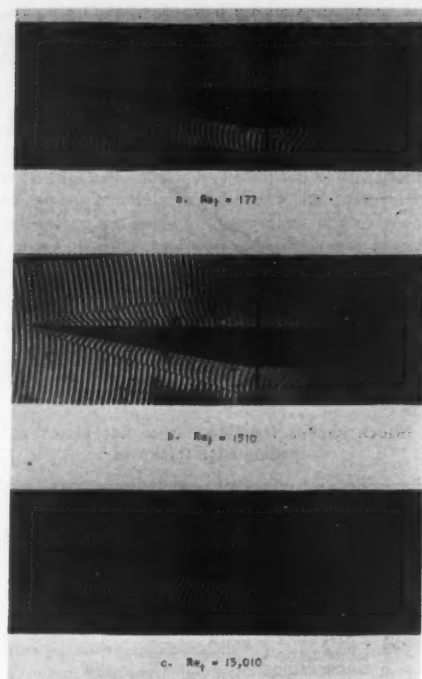
Fig. 5 Interferograms of flat plate mode. $M = 1.27$

flow field is not completely two-dimensional (the model does not span the tunnel) and the fringe shifts are too small to be accurately determined. Only the position of the shock can be determined and quantitatively analyzed.

Surface pressures were measured for the thicker leading edges. Pressure measurements were made on leading edge thicknesses between 1 and 43.8 thousandths of an inch. These measurements were made in two series, the first series at 0.001, 0.004, 0.007 inches leading edge thickness were made in the first nozzle at $M = 10.75$ and 11.8 and P_o of 800, 1000, and 1400 psia. The leading edges were not measured with the microscope and are not known with the same exactness as in the other tests. The second series of tests were for thicknesses between 0.0176 and 0.0438 inches and were made in the second nozzle with the use of the microscope for leading edge measurements. Mach numbers of 11.4, 12.7, and 13.8 were run at $P_o = 800$, 1000, and 1400 psia.

The model used for these measurements was similar to the one shown in Fig. 4, but contained pressure orifices along its top surface. Several orifices were placed off center line to check the tip effects. The tip effects were found to be negligible over the central portion of the plate. A pressure rise was found near the back of the plate which could be traced to the forward propagation of the shocks around the sting support. Only the data which were not influenced by the finite dimensions of the plate are presented. For most of the tests a square leading edge shape was used. A few of the thicker tests were run with a rounded leading edge to find the effects of the detailed leading edge shape.

Analysis

The positions of the shock waves as shown by the interferograms were read on an optical comparitor. These positions, reduced by the leading edge thickness, are shown in Fig. 6. This figure shows that the thicker leading edge shock shapes correlate very well on this plot while the thinner leading edge shapes show some deviation. The difference becomes pronounced below an Re_t of about 4000. These measurements were made in the Mach number gradient shown in Fig. 3. No correction was made to the shock shapes

Fig. 4 Flat plate model
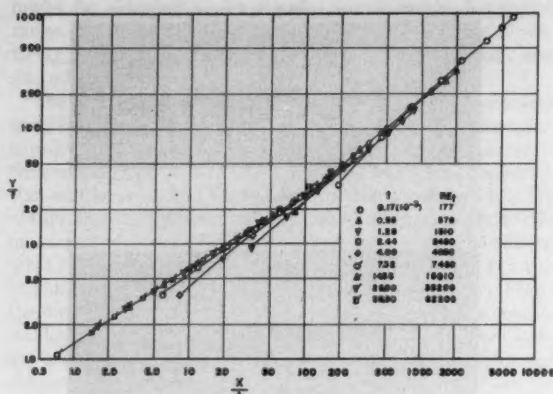
Fig. 6 Shock shapes for flow about flat plates at variable leading edge thickness
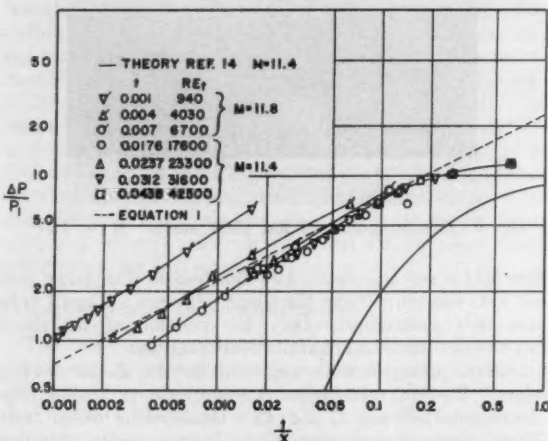
Fig. 7 Surface pressure increment for flow about a flat plate at variable Re_t

so that these should only be compared with each other and are not the shape for flow at constant Mach number. The pressure distributions have also been plotted as a function of t/x. These results have been corrected to a first order, for the Mach number gradient, by using the difference between the pressure on the plate surface and the tunnel empty static pressure at the same point. This nondimensionalized difference is plotted in Fig. 7 for $P_o = 1000$ psia and $M = 11.4$ and 11.8. Only these Mach numbers were used in this plot for the sake of clarity, but these results are typical of all the Mach numbers. No significant difference is found between the square and the round leading edge. From this figure it can be seen that all the pressure data, except for that of 0.001-in. leading edge, correlate quite well. This result could have been anticipated on the basis of the shock shape studies that showed correlation on the t/x basis down to Re_t of about 4000.

Leading edge thicknesses greater than Re_t of 4000 give a flow which is independent of Reynolds numbers. This flow is essentially inviscid and the boundary layer does not have a major influence. This conclusion is borne out by the interferograms which did not show the boundary layer for the thick leading edge, Fig. 5(c). At lower Reynolds number, this correlation no longer holds and boundary layer effects are more important. This result again checks with the interferograms which showed a pronounced boundary layer for thin leading edges, Fig. 5(a).

The shock shape and surface pressure distribution for the inviscid flow regime are caused by the interaction of the detached shock wave and expansion waves from around the

leading edge. The magnitude and downstream persistence of the leading edge effects is surprising. Maximum pressures of the order of 10 to 20 times free stream are reached at about 2 leading edge thicknesses from the leading edge followed by the pressure decreasing as $\sqrt{t/x}$ for $t/x < 0.25$. The following empirical relation has been fitted to the decreasing part of the curve.

$$\frac{\Delta p}{p_i} = 0.016\ M^2\ \sqrt{\frac{t}{x}} \qquad \qquad [1]$$

This relation is based on the data at all Mach numbers and all P_o's. It is shown plotted in Fig. 7.

There seems to be no available theoretical treatment of this problem. Most of the simplified techniques used to calculate hypersonic pressure distributions relate the pressure to the local slope of the body. Theories such as those based on Newtonian flow, shock plus expansion, or tangent-wedge are of this type. All such theories will predict constant pressure over the flat plate surface, so are entirely inadequate for this problem. This problem is an extension of the transonic problem to hypersonic velocities. For this reason it may be called a "tri-sonic" problem and a complete solution must involve the same considerations as the transonic problem complicated by the rotational terms introduced by the curved hypersonic shock wave.

In (14), Bertram has suggested a method for handling this problem. This approach was to use the model shown in Fig. 8. This model avoids the subsonic part of the problem by assuming an attached shock with $M = 1$ directly behind the shock. The results calculated for these assumptions are shown in Fig. 7. The numerical values and shape of the curve differ from the experiments, but at least the model predicts a varying pressure.
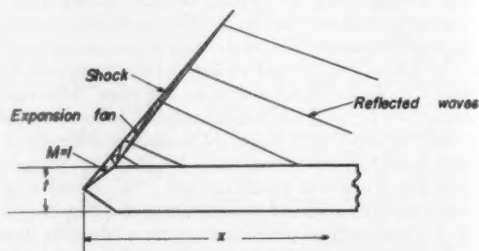
Fig. 8. Simplified flow model

A better understanding of the problem may be had by a study of the hodograph plane shown in Fig. 9 along with the same configuration on the physical plane. If the assumption is made that the flow becomes sonic just before the sharp corner, the contour A represents the flow along the solid boundary. A 90 deg expansion gives infinite Mach number for helium, so zero pressure would be expected just behind the corner. The conditions of the flow along the shock wave are given by the shock polar. The sonic point on the shock is the intersection of the shock polar and sonic circle. Some of the expansion waves from the corner will intersect the sonic line and some will intersect the shock wave. Those intersecting the sonic line will reflect as compressions, while those intersecting the shock will reflect as expansions. The limiting characteristic, the one dividing the waves which intersect the sonic line and those which intersect the shock, is given by contour B. Therefore all the waves starting before B would reflect as compressions and all beyond B as expansions. This reasoning would lead to surface pressure distribution that starts at zero behind the sharp corner, rises through the reflected compressions, and then decreases through the reflected expansions. These compressions and expansions are represented by the part of contour A along the 0 deg axis. The surface condition first moves toward the origin through
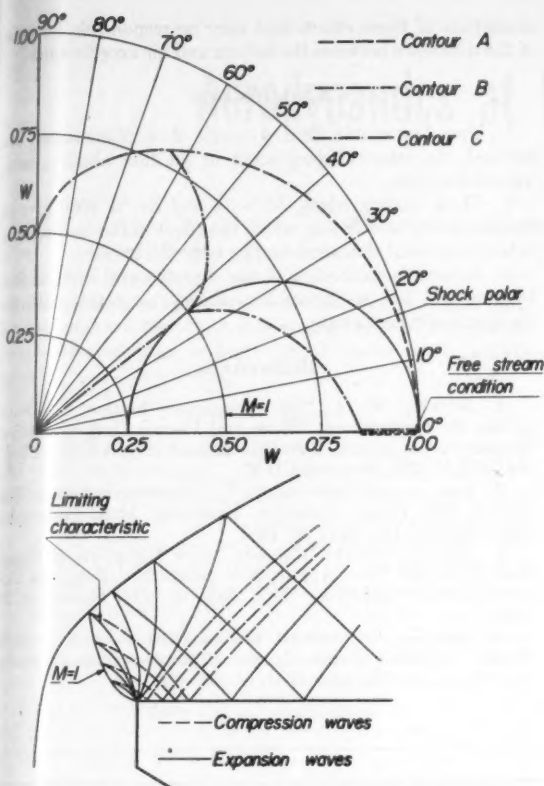
Fig. 9 Flow about a blunt flat plate in the physical hodograph plane

the compression and then away from the origin to the free stream condition. Qualitatively this agrees with the pressure measurements which show a peak value and then a decrease for high values of t/x.

It is interesting to note that the model suggested in (14) gives a considerably different picture on the hodograph plane. In this instance, the conditions on the surface are given by contour C. This model lacks the overexpansion and recompression from the sonic line found for the more correct case.

This description has neglected the vorticity of the flow behind the shock wave. The vorticity will introduce reflections of the expansion and compression waves from the vortex sheets. Therefore, Fig. 9 can be used only as a qualitative picture and an aid in understanding the flow.

Several attempts have been made to account for the effects of viscous forces near the leading edge of a sharp flat plate. All these theories—(1) to (11)—are for the case of an infinitely sharp leading edge. The general approach to this problem has been to take into account the effect on the external flow, and subsequent feedback into the boundary layer, of the change of effective body shape caused by boundary layer displacement thickness. The method used for a first approximation in (2) is relatively simple and shows the general method of attack. The boundary layer displacement thickness for free stream conditions on the surface of the plate is taken as

$$\frac{\delta^*}{x} = 2d_b \frac{M^2\sqrt{C}}{\sqrt{Re_x}} \quad\quad\quad [2]$$

$$C = \frac{\mu_w/\mu_1}{T_w/T_1}.$$

$$d_b = 0.865 \frac{T_w}{M_o^2} + 0.333 \left(\frac{\gamma - 1}{2}\right)$$

at Prandtl number of 1 for the adiabatic case $d_b = 0.599(\gamma - 1)$.

The slope of the boundary layer displacement thickness is

$$\frac{d\delta^*}{dx} = d_b \frac{M^2\sqrt{C}}{\sqrt{Re_x}}$$

This slope is taken equal to the deflection of the free stream. The pressure-angle relation may now be taken from the Prandtl-Mayer or tangent-wedge relation for the free stream. Both of these relations give the same results for the first terms if expanded in series.

$$\frac{p}{p_1} = 1 + \gamma M_1 \frac{d\delta^*}{dx} + \dots \quad\quad [3]$$

Substituting the relation for $d\delta^*/dx$ into Equation [3]

$$\frac{\Delta p}{p_1} = \frac{p - p_1}{p_1} = \gamma d_b \frac{M^2\sqrt{C}}{\sqrt{Re_x}} \quad\quad [4]$$

For the adiabatic case in helium, $\gamma = \frac{5}{3}$

$$\frac{\Delta p}{p_1} = 0.599\,(\gamma)\,(\gamma - 1) \frac{M^3\sqrt{C}}{\sqrt{Re_x}} = 0.66 \frac{M^3\sqrt{C}}{\sqrt{Re_x}}$$

If this expression is assumed to be correct for the infinitely thin leading edge plate, the correct expression for the finite leading edge plate must approach this solution for small Re_t and the inviscid expression for large Re_t. The simplest relation which meets these requirements is a linear combination of Equations [1] and [4].

$$\frac{\Delta p}{p_1} = 0.016\, M^3 \sqrt{\frac{t}{x}} + 0.66\, M^3 \sqrt{\frac{C}{Re_x}}$$

$$\frac{\Delta p}{p_1} = 0.016\, M^3 \sqrt{\frac{t}{x}} \left(1 + 40 \sqrt{\frac{C}{Re_t}}\right) \quad\quad [5]$$

This linear combination neglects the effects of the inviscid flow on the viscous flow, and vice versa. It should be expected to be fairly accurate at the two extremes where one term predominates. From this relation some idea of the relative magnitude of the two terms may be found. The value of Re_t at which both effects are equal is a function of $M(C \sim \overline{M}^{0.7})$. For Mach number of the order of 12, both effects are equal at $Re_t = 400$. All of the surface pressure measurements are at considerably higher Re_t. If the pressure data from the thinner leading edge cases, the only ones in which $\sqrt{C/Re_t}$ is significant, are reduced by the factor $\dfrac{1}{1 + 40\sqrt{C/Re_t}}$ and plotted against t/x, Fig. 10 results. Fig. 10 shows that the 0.001-in. leading edge data correlate fairly well with Equation [5]. The data at 0.004 and 0.007-in. leading edge are overcorrected by Equation [5]. The viscous effect at 0.004 and 0.007-in. leading edge is overestimated
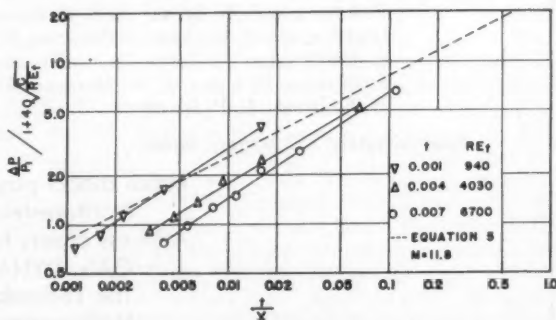
Fig. 10 Reduced surface pressure increment for flow about a flat plate at variable Re_t

by the viscous correction. This result is not unexpected since the Mach number near the surface for the thicker leading edge is much less than free stream. Therefore, the viscous theory, based on free stream Mach number, will overestimate the boundary layer buildup. The relative magnitudes of the viscous and inviscid effects are surprising. In most of the previous work on the leading edge effects on sharp bodies at hypersonic flow, the leading edge inviscid effect has not been recognized. If the Mach number dependence of C is substituted into the relation for surface pressures, the relation becomes

$$\frac{\Delta p}{p_1} = 0.016\, M^3 \sqrt{\frac{t}{x}} \left(1 + \frac{1.45}{M^{0.35}\sqrt{Re_t}}\right) \dots \dots [6]$$

This result indicates that the inviscid leading edge effect is the truly hypersonic one (predominant as $M \to \infty$) and the viscous effect seems to be significant at lower Mach numbers and very sharp leading edges. Because these results have only been derived from experiments over a limited range of Mach number, the accuracy of these extrapolations is open to question. However, the large effect on surface pressures of the leading edge thickness, even at distances greater than 1000 leading edge thicknesses, is most significant.

There are little other experimental data to compare with these results. Some data have been presented in (12) and (14) at Mach numbers of around 6 or 7 in air. These data appear to show much less leading edge inviscid effect than the data herein presented. This result confirms the idea that viscous effect is more important at intermediate speeds. The specific heat ratio probably has some effect on the relative

magnitude of these effects and may be responsible for some of the difference between the helium and air experiments.

Conclusions

1 Two hypersonic fluid dynamic flow phenomena, one inviscid, the other viscous, occur in the flow about a semi-infinite flat plate.

2 Thick leading edges, $M > 10$ and $Re_t > 4000$ give an essentially inviscid flow in which the effect of the leading edge is felt for several thousand leading edge thicknesses.

3 A linear combination of the experimental inviscid flow relation and viscous interaction theory correlates with the sharper leading edge data.

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Aerodynamics at Very High Altitudes[1]

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Research results in the Berkeley low density supersonic wind tunnel are described, covering a range of flow conditions corresponding to flight at altitudes up to eighty miles.

Introduction

A T VERY high altitudes, of the order of 20 miles or more, the earth's atmosphere becomes so rarefied that it no longer behaves as a continuous fluid. Proper account must be taken of the basic molecular structure of the air in the prediction of heat transfer and aerodynamic characteristics. The general field of rarefied gas dynamics, i.e., the mechanics of a gas so rarefied that mean free path effects become important, was discussed by Tsien (1).[3] The terms "slip flow" and "free molecule flow" were introduced to characterize phenomena associated with moderate and extreme rarefaction respectively. A slip flow is a flow in which the mean free path is a small but not negligible fraction of the boundary layer thickness and thus corresponds to values of Mach number M and Reynolds number Re in the range $0.01 < M/\sqrt{Re} < 0.1$, since $M/\sqrt{Re}$ is essentially the ratio of free mean path to boundary layer thickness. A free molecule flow is one in which the mean free path is large compared to the body size, and corresponds to $M/Re > 10$, i.e., to a ratio of mean free path to body dimension of approximately 10 or larger. The slip flow range would apply approximately to flight of a one-foot diameter missile, to take a specific example, at altitudes of the order to 20 to 50 miles, while free molecule flow would obtain at altitudes above 80 miles. Important applications to instrumentation occur at very much lower altitudes. The general regions of flow, together with the ranges of Mach and Reynolds covered by the various experimental programs described subsequently, are indicated in Fig 1.

Two experimental research groups—one at the Ames Laboratory of the NACA under the direction of Stalder and Goodwin, and one at Berkeley under the initial direction of Folsom and Kane—have been carrying on investigations in this area for the past nine years with particular emphasis on applications to high altitude, supersonic flight. Most of the work at Ames has been centered around problems of free molecule flow, while at Berkeley, most of the emphasis has been on the slip flow range. A fairly complete summary report, together with an extensive bibliography, of the progress of research in the field through the summer of 1952 is contained in (2). It is the purpose of this report to describe the principal results obtained at Berkeley during the period 1952–1955 which have application to high altitude aerodynamics. These investigations were carried out in the low density supersonic wind tunnel located at Berkeley and described in (2). Only those investigations which have not yet been published, or which have been published only in the form of project reports, will be discussed here. Other investigations during this period are listed in the bibliography of the present report.
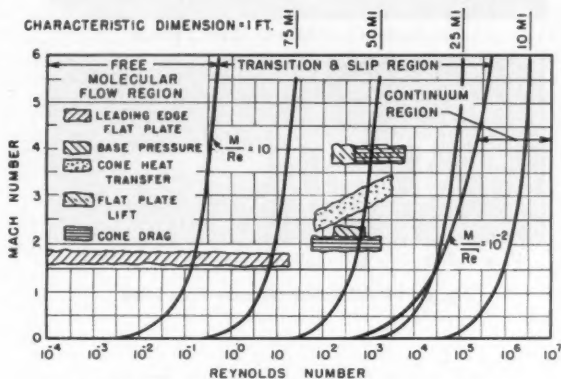
Fig. 1 Mach and Reynolds numbers covered by tests

Of these, special attention should be called to (3) and (4), which present data on the surface pressure distribution on slender cones at a Mach number of 4, and on the skin friction on flat plates at zero angle of attack over a range of Mach numbers from subsonic to 3.5.

Cone Drag

In an inviscid supersonic flow, the surface pressure on a slender cone is constant, providing the Mach number is large enough to insure an attached bow shock wave. In a real viscous fluid, a boundary layer forms, which is particularly thick under the low Reynolds number conditions associated with slip flow. This boundary layer effectively increases the surface curvature of the cone, displacing the stream lines outward. It is to be expected that the surface pressure on a cone will thus be higher near the vertex and decrease along the slant surface, somewhat like the inviscid pressure distribution on an ogive.

In addition to producing a rise in the surface pressure, the thick boundary layer which forms on the surface of a cone under low density conditions also gives rise to a proportionately high skin friction. Ipsen (5, 6) has experimentally measured the total drag of the fore portion of 15 deg half-angle cones over the range $70 < Re < 1600$, $M \sim 2$; $500 < Re < 7000$, $M \sim 4$. Here M and Re are the Mach and Reynolds numbers based on the free stream flow and the cone slant length. A force-sensitive element mounted inside a 2-in. diam cone-cylinder housing was used to measure the total drag force on the floating fore portion of the cone, as indicated in Fig. 2. This total drag had components due to skin friction and also due to the surface pressure rise induced by the boundary layer. The technique did not permit separate evaluation of the two components. The results are presented in Fig. 3, in terms of the increment in the total drag coefficient over that corresponding to inviscid flow. The curves indicated in Fig. 3, are the theoretical predictions obtained by Probstein and Elliott (7), which take account of the induced effect due to the boundary layer. As will be noted, the agreement is quite good, except at the lowest Reynolds numbers where the effects of slip might be expected to begin to be of importance.

1-in. model in position in Mach 4 nozzle
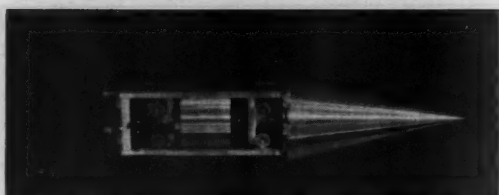
View with balance removed from housing (1-in. model).

Fig. 2   Model set-up for cone-drag tests
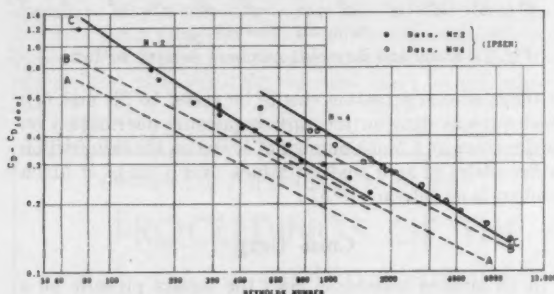
Fig. 3   Drag coefficients for cones with 15-deg semivertex angle

Flat Plate Lift

Normal forces on flat plates at angle of attack have been determined by Tellep (8). The experiments covered the range $228 < Re < 2412$, $2.42 < M < 3.69$ at angles of attack up to 8 deg. Re is the Reynolds number based on the free stream flow and on the length of the plate; M is the free stream Mach number. The results are presented in Fig. 4, in terms of a comparison with the theoretical pressure distribution to be expected on the basis of the hypersonic interaction theory of Lees and Probstein (9). It will be observed that the agreement between experiment and theory is good as regards trend, but that the theory in general underestimates the actual pressure disturbance induced by the boundary layer. This is in accord with experiments at higher pressures conducted by Ferri (10).
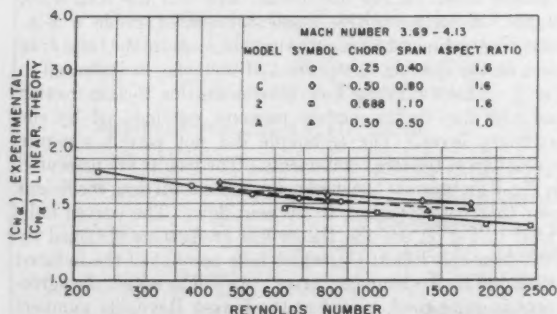
As with the cone drag data of Ipsen, there is little indication of any slip effects. Both the cone and the flat plate are characterized by inviscid constant pressure distributions. To first order, therefore, the pressure gradient at the seam of the boundary layer is zero. Since the effect of slip to first order has been shown (11, 12) to be proportional to this pressure gradient, it is not surprising that slip effects are negligible for these two geometries. Experiments are now under way with a geometry for which the inviscid pressure gradient is not zero, with the expectation that slip effects will be more apparent.

Base Pressure on Cone Cylinders

The pressure on the base of typical cone-cylinder configurations in supersonic flow has been investigated in (13, 14) for the range $Re > 40,000$, where Re is the Reynolds number based on the free stream and the cone-cylinder length. Kavanau (15) has now extended these results to lower Reynolds numbers, $160 < Re < 800$, at $M \sim 2.10$ and $920 < Re < 7400$ for $M \sim 4.0$. He finds considerable variation in the pressure over the base, the pressure at the center being as much as twice as high as the pressure near the edges. His results, in terms of the area mean of the base pressure are presented in Fig. 5, together with the higher Reynolds number results indicated above. No adequate theory now exists for predicting the base pressure under these flow conditions. It is therefore difficult to determine the relative importance of slip-flow effects upon the observed values.
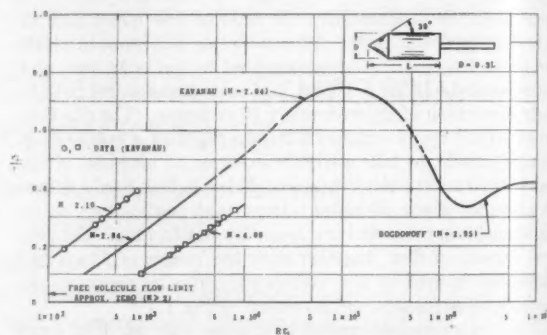
Fig. 5   Base pressure coefficient for cone-cylinder configuration

Cone Heat Transfer

The thermal characteristics of a series of cones at zero angle of attack have been determined in the slip flow range by Drake and Maslach (16). The data covered the ranges $87 < Re < 3270$; $2.16 < M < 3.54$, where M and Re are the Mach and Reynolds number based on free stream flow conditions and the cone slant length. The results are presented in Figs. 6 and 7 in terms of the recovery factor r and the Nusselt number Nu defined by

$$r = \frac{T_{eq} - T}{T_0 - T}$$

$$Nu = \frac{Q}{\rho U c_p A (T - T_{eq})}$$

where T, T_{eq} and T_0 are the stream temperature, the cone equilibrium temperature, and the adiabatic stagnation temperature, respectively, Q is the heat flux to the cone, ρ, U, and c_p free stream density, velocity and specific heat at constant pressure, and A is the cone surface area. The increase in the recovery factor with decreasing Reynolds number is interpreted as being due to the large dissipation of energy by fric-

Fig. 4   Ratio of experimental normal force curve slope to theoretical normal force curve slope vs. Reynolds number
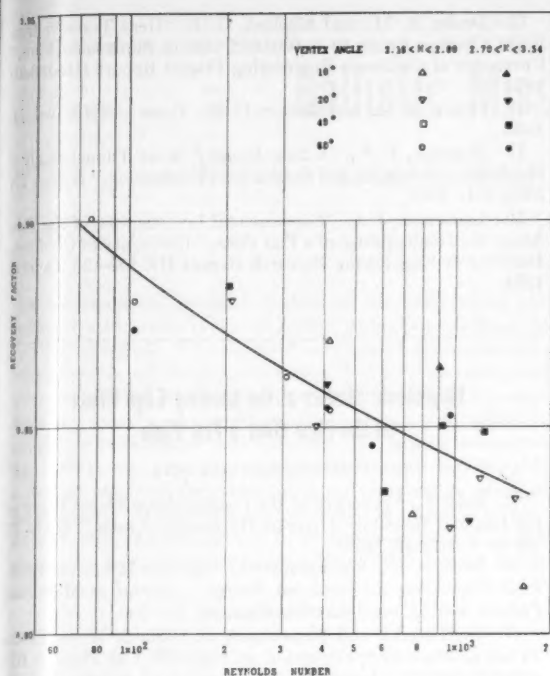
Fig. 6 Thermal recovery factors for cones in supersonic flow
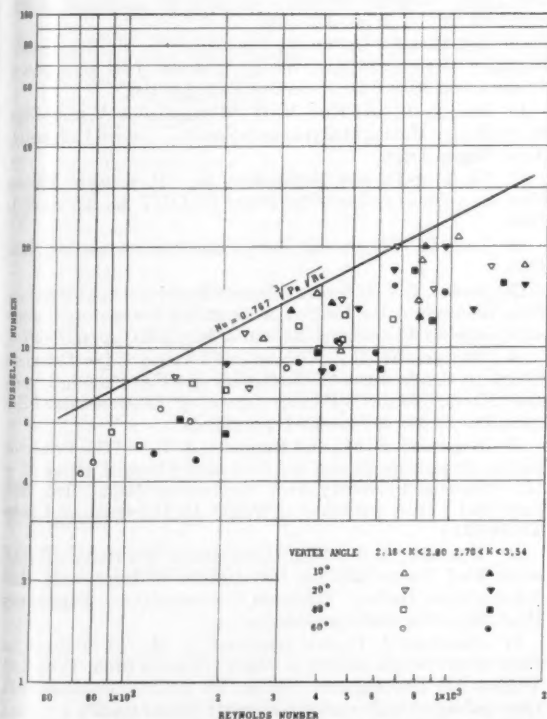
Fig. 7 Convective heat transfer coefficients for cones in supersonic flow

tion. The trend indicates a recovery factor greater than unity for somewhat lower pressures. Such values have been observed for spheres (17) and constitute an important effect for high altitude flight considerations. The heat transfer rate, on the other hand, decreases with increasing rarefaction. This is probably due to the effective thermal film resistance introduced by the temperature jump at the surface.

Survey of Boundary Layer Leading Edge Region

The thermal recovery factor of any convex body in free molecule flow is a rather sensitive function of the Mach number over the range $0.5 < M < 3.5$. The equilibrium temperature of a wire whose diameter is small compared to the mean free path and which is stretched across the test section of a wind tunnel thus depends on the Mach number and the local stagnation temperature. A "free molecule probe" of this sort has been used by Sherman (18) to determine the internal structure of a normal shock wave. The probe has been used by Laurmann (19), to explore the flow field in the immediate vicinity of a flat plate in supersonic flow. Only a qualitative picture can be obtained, since both the Mach number and the stagnation temperature vary in an interrelated way. How-
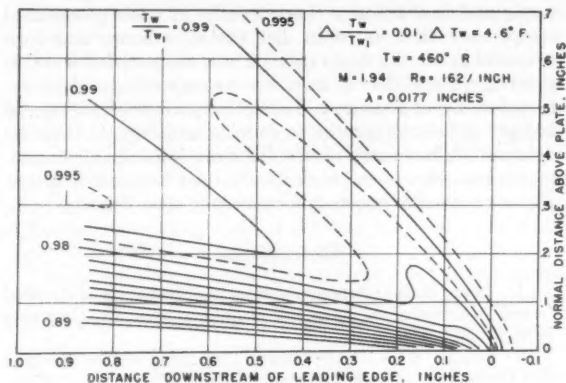
Fig. 8 Wire temperature isotherms $M \sim 2$, mean free path, $\lambda = 0.0177$ in.
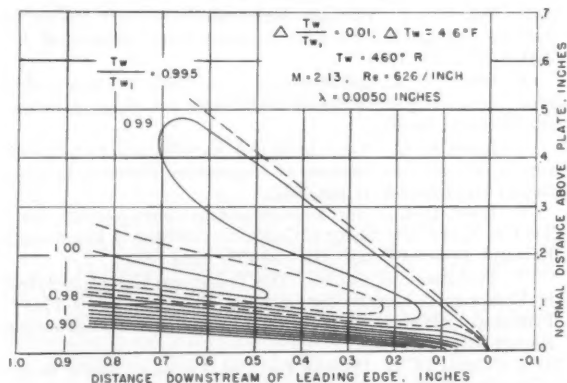
Fig. 9 Wire temperature isotherms $M \sim 2$, mean free path, $\lambda = 0.0050$ in.
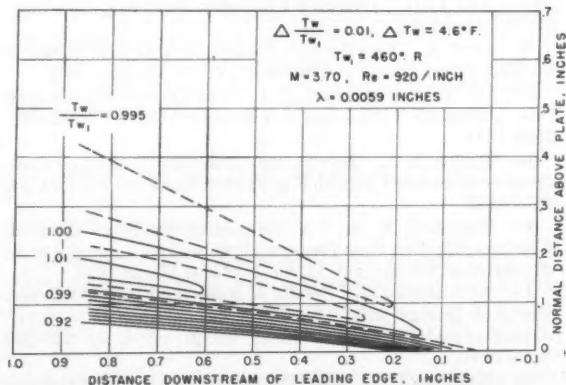
Fig. 10 Wire temperature isotherms $M \sim 4$, mean free path, $\lambda = 0.0059$ in.

ever, the general, over-all nature of the flow field can be inferred. A serious question for the hypersonic boundary layer interaction theory is the strength of the shock at the leading edge. The existence of a region of slip here could have a very great influence far downstream. The results are presented in Figs. 8, 9, and 10 in the form of contours through the flow field of constant wire temperature. These clearly show the shock and boundary layer regions. The possibility of slip at the higher Mach number is strongly suggested. These tests were carried out using "flat plates" in the form of ribbons only a small fraction of a mean free path thick. This work is now being extended to higher Mach numbers.

Conclusions

Empirical data are now available for predicting aerodynamic and heat transfer characteristics of basic geometrical components such as cones, flat plates, spheres, and cone cylinders in the slip flow range. These characteristics are, in general, quite different from the corresponding values obtained at higher pressures. Theoretical analyses, however, are not yet sufficiently precise to provide an adequate basis for determining how much of this difference is ascribable to noncontinuum phenomena such as slip, and how much to low Reynolds number supersonic continuum flow behavior.

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Ballistic Missile Performance[1]

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A semianalytic method is derived for calculating the range of one-stage, ballistic, rocket propelled, surface-to-surface missiles. The method consists of obtaining certain analytic solutions, supposing flight in vacuum, and then correcting these solutions for the effect of drag. Both the velocity at thrust cutoff and the range following cutoff are handled in this manner. An analog computer was used to compute the drag corrections and also the altitude and range at thrust cutoff for a large range of input parameters. The results are presented as working formulas and plots suitable for use in preliminary design studies of missiles having ranges of from 50 to 500 nautical miles.

Nomenclature

A = area of cross section
a = A/W_o
C_D = drag coefficient based on area of cross section
$C_D{}^*$ = drag coefficient for $M = 2$
D = drag
F = function plotted in Fig. 5
G = function plotted in Fig. 7
g = acceleration due to gravity
h = altitude
I = τ/r = specific impulse
M = Mach number
r = $-(1/W_o)\cdot(dW/dt)$ = burning rate
R = radius of the earth
t = time
T = thrust
V = velocity
ΔV_D = loss of cutoff velocity due to drag
W = weight
X = range after cutoff for vacuum trajectory over spherical nonrotating earth
ΔX_D = range loss due to drag after cutoff
η = angle shown in Fig. 2
θ = flight path angle with local horizontal
ρ = air density
ν = W_o/W_{co}
τ = T/W_o
ϕ = $\dfrac{\cos\theta_{co} - \cos\theta_o}{\theta_o - \theta_{co}}$

Subscripts

o refers to conditions at launching
co refers to conditions at the end of burning

Introduction

ONE measure of the performance of a ballistic surface-to-surface missile is its maximum range with a given payload. A simple method for calculating this range has been devised. The type of missile considered is single stage and powered by a rocket motor.

The method is intended principally for use in preliminary design work. It should be of particular use in design comparison studies, economic studies, and investigation of the

trade-off between payload and range. In such applications the effect of changes in the missile parameters is of greater interest than the exact magnitude of the range itself. This calls for a simplified procedure for calculating range. In order to meet this objective, it is necessary to introduce several approximations into the work. Sufficient accuracy is to be maintained for the intended uses.

For the purpose of calculating range the missile may be described by a set of parameters. The basic parameters considered are: 1. thrust ratio, τ; 2. mass ratio, ν; 3. specific impulse, I; 4. area ratio, a; 5. drag function, C_D vs. M.

The ballistic missile derives the major part of its range after thrust cutoff. For the case of trajectory in vacuum over spherical nonrotating earth, a solution in closed form is obtained for maximum range after cutoff.[4]

The range after cutoff is a function of the cutoff velocity and altitude. The dominating factor, cutoff velocity, can be solved analytically if drag is excluded.

The drag terms are not susceptible to analytic treatment. This difficulty was overcome by using an analog computer. The computer also provided cutoff altitude and range.

The ballistic trajectory following cutoff is considered first, in order to establish a certain boundary condition needed for programming the burning phase, which is the flight path angle at cutoff.

Assumptions

The following principal assumptions are made:

1 The earth is spherical and nonrotating.
2 Launching and impact are at sea level.
3 The missile follows a particle trajectory.
4 Thrust and burning rate are constant.
5 The missile is launched vertically, and during the burning phase $d\theta/dt = $ const.
6 The drag corrections are small compared with the quantities to which they are applied.
7 Earth curvature may be ignored during the burning phase and also in obtaining the drag correction to range after cutoff.

Thrust and specific impulse, connected through the relation $I = \tau/r$, are to be represented by chosen mean values in recognition of the fact that rocket thrust varies somewhat with altitude. Any secondary propellants must be accounted for in r, the total discharge rate, and consequently in the specific impulse as defined here.

Ballistic Trajectory Following Cutoff

Trajectory and Range

The vacuum trajectory of an object moving within the gravitational field of spherical nonrotating earth, and having less than the energy required for escape, is a Keplerian ellipse (3).[5] One of the foci of the ellipse is at the center of

[4] It has been brought to the attention of the authors that similar solutions have been obtained (1, 2) for the limiting condition $h_{co} = 0$.
[5] Numbers in parentheses indicate References at end of paper.
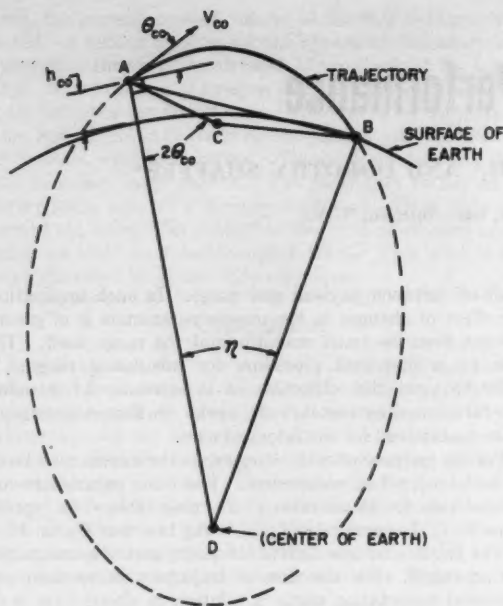
Fig. 1 Vacuum trajectory over spherical earth. Ellipse with foci at C and D

the earth (see Fig. 1). The length of the majorax is L is given by

$$L = \frac{gR^2}{gR - [\frac{1}{2}V_{co}^2 + gh_{co}/(1 + h_{co}/R)]} \\ = \frac{R + h_{co}}{1 - (1 + h_{co}/R)V_{co}^2/2gR} \Bigg\} \quad \dots \dots [1]$$

Let it be assigned to maximize the range following cutoff, $X = R\eta$, for particular values of V_{co} and h_{co}. This range is a maximum when the length AB (Fig. 1) is maximized. The sum of the focal radii AC and CB is given by

$$AC + CB = 2L - 2R - h_{co} \dots \dots [2]$$

which is fixed for chosen values of V_{co} and h_{co}. This being so, the length AB is obviously a maximum when the second focus C lies on a line joining points A and B.

The three sides of the triangle ABD are now known for the condition of maximum range, and from the cosine law the angle η_{max} can be solved. The resulting equation can be written in the form

$$\eta_{max} = 2 \sin^{-1} \frac{c}{1 - c} \sqrt{1 + h_{co}/cR} \dots \dots [3]$$

in which

$$c = \frac{V_{co}^2}{2gR} \left(1 + \frac{h_{co}}{R}\right) \dots \dots [4]$$

Flight Path Angle at Cutoff

Through use of the property that the focal radii intersecting at a point on an ellipse make equal angles with the tangent at that point, it can be demonstrated that the angle DAC, Fig. 1, is equal to $2\theta_{co}$. The cosine law may be applied to the angle DAB to provide a solution for the optimum value of θ_{co}; that is, the value that maximizes range for given V_{co} and h_{co}. The solution yields

$$\theta_{co_{opt}} = \sin^{-1} \sqrt{\frac{c}{1 - c} \left[\frac{1 - c(2 + h_{co}/R)}{h_{co}/R + c(2 + h_{co}/R)}\right]} \dots [5]$$

Approximations

For ranges not exceeding 500 nautical miles, $\eta/2 < 5$ deg, so that with sufficient accuracy, $\sin \eta/2 = \eta/2$ may be used to simplify Equation [3]. The maximum range following cutoff, for trajectory in vacuum, is then given with good accuracy by

$$X = \frac{2cR}{1 - c} \sqrt{1 + h_{co}/cR} \dots \dots [6]$$

Furthermore, since $h_{co}/R << 1$, this term may be neglected in Equation [4] if desired.

Equation [5] may be approximated upon expanding the equation and ignoring terms of the order c^2, $c(h_{co}/R)$, $(h_{co}/R)^2$ and higher. Then, since $h_{co}/R << 1$, c is replaced by $V_{co}^2/2gR$, with the result

$$\sin^2 \theta_{co_{opt}} \approx \frac{1}{2} \left(1 - \frac{V_{co}^2}{2gR} - \frac{gh_{co}}{V_{co}^2}\right) \dots \dots [7]$$

Equation [7] may be compared with the expansion of $\sin^2 \theta$ about $\pi/4$, retaining only the first order terms in the expansion, and it is seen that

$$\theta_{co_{opt}} \approx \frac{\pi}{4} - \frac{V_{co}^2}{4gR} - \frac{gh_{co}}{2V_{co}^2} \dots \dots [8]$$

An alternative form is as follows

$$\theta_{co_{opt}} \approx \frac{\pi}{4} - \frac{X}{4R} - \frac{h_{co}}{2X} \dots \dots [9]$$

These last equations put the desired cutoff angle at something less than 45 deg. In the cases covered by the work that follows it would fall between 39 deg and 43 deg. To facilitate the burning phase analysis a cutoff angle of 41 deg was chosen to handle all cases.

The remainder of the work will be directed toward the calculation of a sensible maximum total range. The calculation of a true maximum would involve extensive burning phase analysis plus the use of variational techniques. Such detailed analysis is not in line with, and would not contribute significantly to, the present objective.

The range X is to be calculated from Equation [6] regardless of the fact that the burning phase is assumed to be terminated in all cases at a flight path angle of 41 deg. If the total range is near maximum, the combined range-producing potential of V_{co}, h_{co}, and X_{co} is insensitive to minor changes in the burning phase history. An example of this statement can be seen in Fig. 2, which is discussed in the next section.

Burning Phase Trajectory

Programming

Since ballistic missiles are being dealt with, one would be inclined to assume a gravity turn path for the burning phase. However, this would complicate the analysis. For every set of missile parameters several trials would have to be made, each time varying the initial launch angle, in order to arrive at the prescribed boundary condition on flight path angle at cutoff. It was necessary to avoid this.

Instead, the missile is considered to be launched vertically, and caused to turn at a constant rate during the burning phase. The effect of this programming was checked against a maximum range gravity turn trajectory. The result, shown in Fig. 2, was obtained with an analog computer. Since the difference in range could not even be detected, the chosen programming is evidently a good compromise in so far as it simplifies the analysis.

It is intended that the total range to be calculated in the present method be understood to be essentially the maximum range of a missile following a zero lift, or ballistic, trajectory throughout flight.
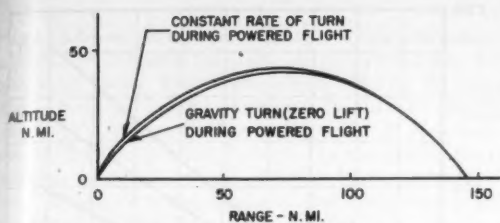
Fig. 2 Comparison of trajectories having different programming during powered flight

Fig. 3 Assumed variation of drag coefficient with Mach number

Cutoff Velocity

The cutoff velocity is obtained upon integrating the equation of motion over the time of burning. The equation of motion in the direction of flight is

$$T - D - W \sin \theta = (W/g)(dV/dt) \dots \dots [10]$$

Under the assumptions that thrust, burning rate, and rate of turn are constant, the integration furnishes a formula for the cutoff velocity.

$$V_{co} = \frac{gT}{\tau W_o} \log \frac{W_o}{W_{co}} - \frac{\cos \theta_{co} - \cos \theta_o}{\theta_o - \theta_{co}} g t_{co} - \int_0^{t_{co}} \frac{gDdt}{W_o(1 - rt)}$$
$$\dots \dots [11]$$

$$= gI \log \nu - g\phi t_{co} - \Delta V_D \dots \dots \dots \dots \dots \dots \dots [12]$$

The burning time is obtained from

$$t_{co} = \frac{I}{\tau}\left(1 - \frac{1}{\nu}\right) \dots \dots \dots \dots \dots [13]$$

Computer Operations

The unsolved term in the equation for cutoff velocity must be solved. The cutoff range and altitude must be calculated. Finally, a correction for air resistance must be applied to the range after cutoff.

This was all done with the aid of an analog computer. The computer was wired to solve the complete trajectory of a ballistic missile in the presence of the earth's atmosphere. The curvature of the earth was neglected. The atmospheric functions, speed of sound and air density, were approximated as analytic functions of altitude, so that they could be generated using ordinary machine components.

The air density was represented by two third-degree polynomials in altitude. These were obtained as a least-squares fit to data for the NACA Standard Atmosphere. The first polynomial supplied the atmospheric density ratio from sea level to 100,000 ft. At 100,000 ft the second polynomial was switched in, supplying the density ratio between 100,000 ft and 200,000 ft. Above 200,000 ft the density was assumed zero.

The speed of sound was approximated as decreasing linearly with altitude, from a value of 1116 ft/sec at sea level to a value of 972 ft/sec at 35,000 ft above which it was assumed to remain constant.

Drag Function

A drag curve made up of analytic elements, as shown in Fig. 3, was assumed. This curve is believed to be a reasonable approximation to the typical drag curve of a cylindrical body with pointed nose and stabilizing fins.

Despite the arbitrarily assumed burning phase programming, neither lift forces nor drag due to lift were considered in the analog computer operations. Reasons for this were covered in the discussion of Fig. 2. The drag was computed through the equation

$$D = C_D \frac{1}{2} \rho V^2 A \dots \dots \dots \dots \dots [14]$$
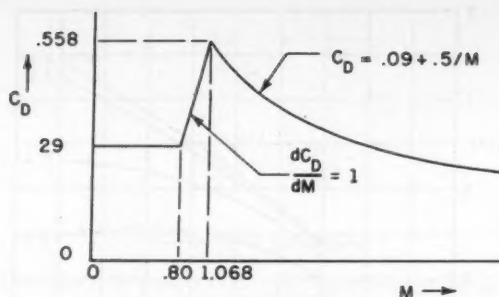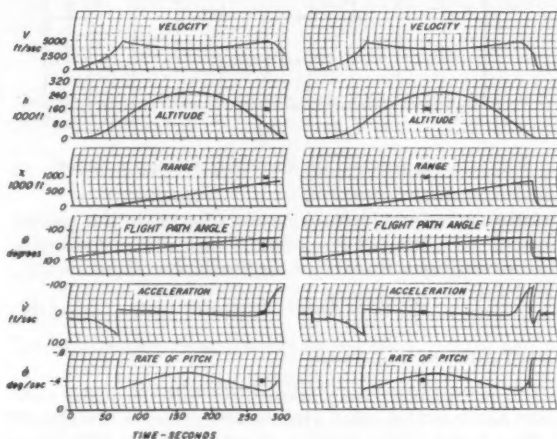
Fig. 4 Hand calculation (left) and REAC calculation (right) for flat earth performance of a hypothetical one-stage missile having the characteristics: $I = 235$ sec; $\nu = 2.71$; $\tau = 2.12$; $\alpha = 0.00082$ ft.²/lb.; C_D vs. M from Fig. 3.

As a check on the operations of the computer, Fig. 4 presents for comparison the results of an analog computer calculation and a hand calculation made using the same input parameters.

Parameter Bounds

Using the analog computer, a large number of runs were made covering the burning phase. The parameters that were varied, and their bounds, are

Thrust ratio	$1.5 \leq \tau \leq 4.0$
Mass ratio	$2.0 \leq \nu \leq 4.8$
Specific impulse	$200 \leq I \leq 235$ sec
Area ratio	$0.0005 \leq a \leq 0.0015$ ft²/lb

Burning Phase Drag

The unsolved term, ΔV_D, in Equation [12], which is the decrease of cutoff velocity due to drag, is given by

$$\Delta V_D = \int_0^{t_{co}} \frac{ga C_D \frac{1}{2} \rho V^2}{1 - rt} dt \dots \dots \dots [15]$$

The integral in Equation [15] was computed over a wide range of parameters. Function F, defined by

$$F = \frac{1}{C_D{}^*} \int_0^{t_{co}} \frac{g C_D \frac{1}{2} \rho V^2}{1 - rt} dt \dots \dots \dots [16]$$

in which $C_D{}^*$ is the drag coefficient for a Mach number of 2.0, can be plotted as a function of thrust ratio τ and mass ratio ν alone, since the calculated results were found to be insensitive to changes in the remaining parameters. The $C_D{}^*$ behaves as a mean value of drag coefficient, which was the desired effect.

The plot for function F is shown in Fig. 5. This and the re-
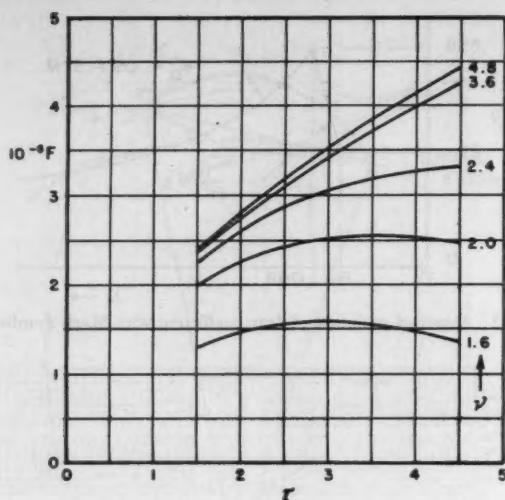
Fig. 5 Drag function for correcting velocity at cutoff

maining figures provide working plots for use in the present
method. The value of F obtained from Fig. 5 is inserted in
the following equation to solve the final term in Equation [12].

$$\Delta V_D \text{ (ft/sec)} = aC_D*F \dots\dots\dots\dots [17]$$

Cutoff Altitude and Range

The burning phase calculations made with the analog com-
puter also provided cutoff altitude and range, shown plotted
in Figs. 6 and 7 as a function of cutoff velocity for various
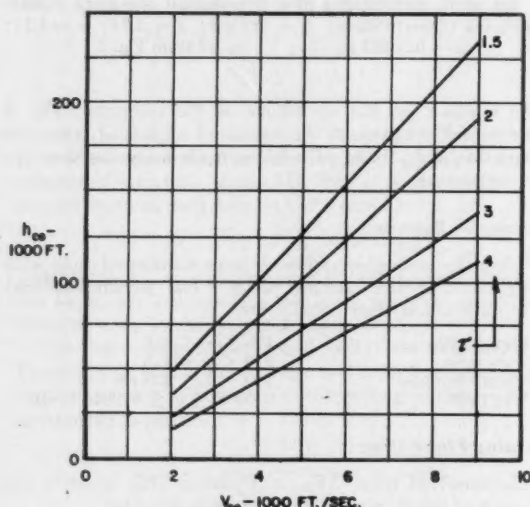
Fig. 6 Altitude at cutoff

values of parameter τ. The plotted data were observed to
be insensitive to changes of parameters ν and a. A further
dependence of h_{co} and X_{co} on specific impulse is not reported in
the plots, because of its rather small consequence. An ap-
proximate correction based on the original plotted data may
be applied to the values of h_{co} and X_{co} read from Figs. 6 and 7,
through the formula $\Delta h_{co}/h_{co} = \Delta X_{co}/X_{co} = 0.8\Delta I/I_o$, in
which $I_o \equiv 220$ sec.

Drag After Cutoff

It remains to correct the range following cutoff for the
effect of air resistance. This was also done using the analog
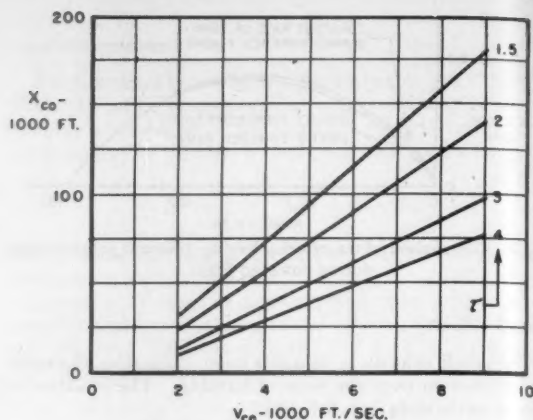
Fig. 7 Range at cutoff

computer. A method somewhat similar to that handling
burning phase drag was employed. The range was computed,
both in atmosphere and in vacuum, for a number of combi-
nations of cutoff velocity and altitude, and three values of area-
to-cutoff weight ratio, νa. It was observed that the difference
between these range computations, which is the loss of range
due to air resistance after cutoff, is proportional to νa. A further
factor, drag coefficient just after cutoff, C_{Dco}, is introduced.
The reason for this particular choice of drag coefficient is that
the drag losses immediately following cutoff account for the
greater part of the range reduction.

Function G was calculated through an inversion of the equa-
tion

$$\Delta X_D \text{ (ft)} = \nu a C_{Dco} G \dots\dots\dots\dots [18]$$

and then plotted as shown in Fig. 8. The range correction can
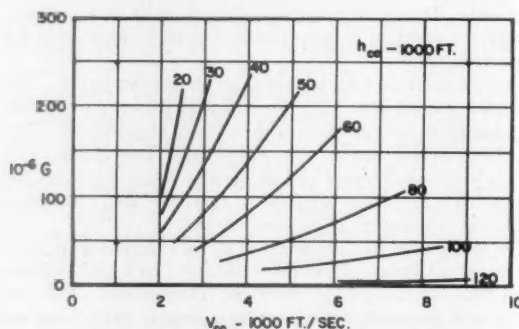
Fig. 8 Drag function for correcting range after cutoff

now be reconstructed upon inserting into Equation [18] the
value of G obtained from Fig. 8. In determining C_D* and
C_{Dco}, account may be taken of any change in drag due to the
application or termination of thrust.

Application and Evaluation

Parameter Flexibility

The range of specific impulse values admissible under this
method is somewhat wider than the bounds that were shown
above. The reason is that the primary effect of specific
impulse is handled explicitly in the propulsion term of
Equation [12]; secondary effects are unimportant.

There is also reason to believe that the procedure for
handling drag permits the use of drag functions differing in
detail from the curve shown in Fig. 3. This is found in the
results of the burning phase calculations using various values

of the area-to-weight ratio, a. Changes in this area parameter cause similar changes in the drag. As the over-all drag level is changed, it causes the burning-phase histories of velocity, altitude, and thus dynamic pressure to be altered. The effect is similar to distorting the drag curve itself. It was observed, however, that the function F, needed to reconstruct the drag correction to cutoff velocity, remained insensitive to this effect. This leads to the conclusion that moderate changes in the shape of the drag curve can be tolerated.

Range Limits

At decreasing ranges the effect of drag rises in relative importance. If the major part of the drag loss occurs at transonic or especially subsonic speeds, the method as presently constituted cannot accurately predict the drag effects. For this reason caution should be used in applying this method to the calculation of ranges less than 50 miles.

For ranges exceeding 500 miles, the optimum cutoff angle begins to swing too far below 41 deg for the present results to be valid.

Accuracy

The original data have been examined to provide an idea of the accuracy of the method. Within the parameter bounds previously tabulated it is estimated that the errors contained in the values read from Figs. 5 to 8 will seldom exceed the following:

Function F, Fig. 5	$\pm 0.25 \times 10^6$ units error
h_{co}, Fig. 6	$\pm 5\%$ error (when corrected for I)
X_{co}, Fig. 7	$\pm 5\%$ error (when corrected for I)
Function G, Fig. 8	$\pm 10\%$ error

It is estimated that use of C_D^* as directed will not usually result in an error in excess of ± 10 per cent for drag curves other than that assumed here.

The total range is affected directly by errors in X_{co} and ΔX_D. The effect on range of errors in V_{co} and h_{co} can be determined by writing the first order approximation for Equation [6]

$$X \approx \frac{V_{co}^2}{g} + h_{co} \dots \dots \dots \dots \dots [19]$$

from which

$$dX = 2(V_{co}/g)dV_{co} + dh_{co} \dots \dots \dots \dots [20]$$

The error in h_{co} thus produces an equal error in range, while the range error produced by error in V_{co} is $2V_{co}/g$ times the error in V_{co}.

Calculation Procedure

In making use of the present method the parameters τ, ν, I, a, and the function C_D vs. M must be known. The steps in calculating range are:

1 Calculation of velocity at thrust cutoff:

$$V_{co} \text{ (ft/sec)} = 32.2I \log \nu - 28.4 \frac{I}{\tau}\left(1 - \frac{1}{\nu}\right) - aC_D^*F \dots [21]$$

2 Determination of altitude and range at cutoff. The values read from Figs. 6 and 7 can be rendered more accurate by correcting them for specific impulse, as previously noted.

3 Summation of the components of the range:

$$\text{total range} = X_{co} + \frac{2cR}{1-c}\sqrt{1 + h_{co}/cR} - \nu aC_{Dco}G \dots [22]$$

in which

$$c = \frac{V_{co}^2}{2gR}\left(1 + \frac{h_{co}}{R}\right)$$

Numerical Example

The hypothetical missile for which the parameters are given in Fig. 4 may be used to illustrate the application of the method. From Fig. 3, $C_D^* = 0.34$; and from Fig. 5, $F = 2.75 \times 10^6$. The cutoff velocity is now solved using Equation [21]:

$$V_{co} = 4780 \text{ ft/sec}$$

Using Figs. 6 and 7, and making corrections for specific impulse:

$$h_{co} = 99,000 \text{ ft}$$
$$X_{co} = 72,000 \text{ ft}$$

The Mach number at cutoff is $4780/972 = 4.9$. Figs. 3 and 8 are entered to find $C_{Dco} = 0.19$; $G = 28 \times 10^6$.

Before calculating the range, the function c must be solved:

$$2cR = 714,000 \text{ ft}; \quad c = 0.0170$$

The total range is found from Equation [22]:

$$\text{Total range} = 882,000 \text{ ft or } 145 \text{ n.mi.}$$

The optimum value of flight path angle at cutoff is, from Equation [8], 41.0 deg.

The hand calculations for the performance of this missile yielded the following results:

$$V_{co} = 4800 \text{ ft/sec}$$
$$h_{co} = 100,800 \text{ ft}$$
$$X_{co} = 69,400 \text{ ft}$$
$$\text{Total range} = 146 \text{ n.mi. (corrected for earth curvature)}$$

The maximum expected errors in the example worked out above are as follows: V_{co}, 70 ft/sec; h_{co}, 5000 ft; X_{co}, 3600 ft; ΔX_D, 1200 ft. The range error produced by the 70 ft/sec error in V_{co} is 20,800 ft, which is here by far the most important error.

Under the conservative assumption that the errors in h_{co} and X_{co} are mutually consistent, the maximum expected total range error for the example, taken as the rms sum of 8600, 1200, and 20,800 ft, comes to 22,500 ft or 3.7 n.mi. Had an allowance for 10 per cent error in C_D^* been made to provide for different drag characteristics, the error in V_{co} would be $\sqrt{(70)^2 + (77)^2} = 104$ ft/sec, and the total expected range error would be increased to 5.3 n.mi.

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Effect of Chemical Reactions in the Boundary Layer on Convective Heat Transfer[1]

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An analysis is made of the relative change in heat-transfer rate resulting from chemical reactions in the boundary layer for either turbulent or laminar flow. Consideration is given to reactions both in the gas phase and on the wall. It is shown that, for gas phase reactions, if $D = \kappa$ (where D is molecular diffusivity and κ thermal diffusivity) or if reaction occurs in which both D and κ are much less than ϵ (the turbulent-exchange coefficient), the relative change in heat flux is given merely in terms of thermodynamic properties alone. For this case, the use of a "nondissociated" temperature representing the total enthalpy of the system is found to be useful.

Nomenclature

c	=	isobaric specific heat
C	=	isobaric molar heat capacity
D	=	molecular diffusivity
F	=	molar reaction rate per unit area of surface
M	=	molecular weight
n	=	molar concentration
N	=	mole fraction
Pr	=	Prandtl number
q	=	heat flux per unit area
Q	=	heat of reaction per mole of diffusing species ($-\Delta H$ of reaction)
$\mathfrak{R}$	=	molar reaction rate per unit volume
R	=	universal gas constant
Sc	=	Schmidt number
T	=	temperature
y	=	coordinate of heat flow
y^*	=	dimensionless distance parameter $= (y/\nu)\sqrt{\tau/\rho}$
Z	=	rate of molecular collisons per unit surface
α	=	accommodation coefficient for surface reaction (cf. Eq. 14)
β	=	dimensionless constant (cf. Eq. 15)
ϵ	=	turbulent-exchange coefficient
κ	=	thermal diffusivity
λ	=	thermal conductivity
μ	=	viscosity
ν	=	kinematic viscosity
ρ	=	density
τ	=	shear stress
$(\ )_0$	=	wall ($y = 0$)
$(\ )_1$	=	edge of laminar sublayer ($y^* = 5$)
$(\ )_2$	=	outer edge of intermediate layer ($y^* = 30$)
$(\ )_e$	=	property including turbulent contribution
$(\ )_i$	=	diffusing species
$(\ )_m$	=	log mean of property in film
$(\ )'$	=	condition without chemical reaction
$(-)$	=	average property in boundary layer
$(\ )^0$	=	fictitious free-stream property calculated in the absence of chemical dissociation

I Introduction

IN THE combustion processes which yield a high concentration of chemically active species such as free radicals and atoms, the heat transfer between the free stream and the wall may be significantly altered by chemical reaction within the system. A simplified analysis is given of the relative change in heat flux from a hot moving gas stream to a cooled surface as a result of chemical reactions in the boundary layer and on the wall.

The assumptions and approximations incorporated in this treatment are as follows: (a) The gradients of the quantities considered along the flow line parallel to the wall are negligible with respect to gradients normal to the wall (y direction). (b) The coefficients of turbulent exchange for viscosity, diffusivity, and conductivity (eddy quantities) are equal. (c) The boundary layer thickness does not change significantly as a result of heat release; i.e., the Prandtl and Schmidt numbers remain essentially constant (1) and (2).[4] (d) Steady state has been established, and pressure is uniform within the system. (e) The mole fraction of reactive species is low if more than one such species is considered. (f) The cold-wall temperature T_0 is fixed at a relatively low value by coolant flow. (g) Thermal diffusion of species is neglected. For species of equivalent molecular weight, this effect is known to be small. In the case of species with molecular weight much different from that of the mixture (such as H atoms) the inclusion of thermal diffusion would still act only as a correction in the present case and is therefore neglected to the degree of approximation employed in this paper. (h) Radiation effects are not considered.

II Derivation of Equations

Using von Kármán's treatment of the analogy between fluid friction and heat transfer (3), it is assumed that the boundary layer contains a laminar sublayer of thickness $y^* = 5$ and an intermediate layer (buffer layer) in the interval $5 < y^* < 30$. In the sublayer, the transport properties are given solely by the molecular quantities $\nu = \mu/\rho$, $\kappa = \lambda/c\rho$, and D. In the intermediate region, these quantities increase linearly with distance as a result of turbulence. The coefficient of turbulent exchange ϵ (3), also called exchange coefficient (4), represents the turbulent contribution to the transport quantities ν, κ, and D (Fig. 1). If the effective transport parameters are denoted by $\kappa_e = \kappa + \epsilon$ and $D_e = D + \epsilon$ to represent appropriate values in boundary layer, the conservation equations for heat and mass may be written as follows

$$\frac{d}{dy}\left(c\rho\kappa_e \frac{dT}{dy}\right) + Q\mathfrak{R} = 0 \quad\ldots\ldots\ldots\ldots [1]$$

$$\frac{d}{dy}\left(\frac{D_e n}{1 - N_i}\frac{dN_i}{dy}\right) - \mathfrak{R} = 0 \quad\ldots\ldots\ldots\ldots [2]$$

where Q is the heat released in the recombination of 1 mole of the labile species ($-\Delta H$ of reaction), $\mathfrak{R}$ is the reaction rate expressed in moles per unit volume per sec, ρ is the density, n
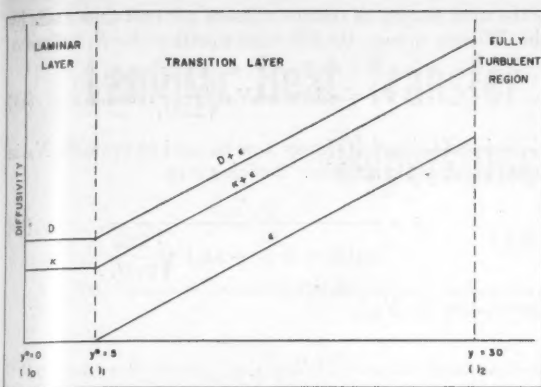
Fig. 1 Variation of exchange coefficient and effective diffusivities in the boundary layer

is the molar density, and N_i is the mole fraction of free radical. Elimination of $\Re$ between Equations [1] and [2] results in

$$\frac{d}{dy}\left(c\rho\kappa_e\frac{dT}{dy}\right) + Q\frac{d}{dy}\left(\frac{D_e n}{1-N_i}\frac{dN_i}{dy}\right) = 0 \ldots\ldots [3]$$

Since the heat of reaction Q is essentially constant, a first integral of Equation [3] between the limits $y = 0$ and $y = y$, gives directly the heat-transfer rate per unit area of wall surface

$$q = c\rho\kappa_e\frac{dT}{dy} + \frac{QD_e n}{1-N_i}\frac{dN_i}{dy} \ldots\ldots\ldots [4]$$

Division by $c\rho\kappa_e$ and integration between $y = 0$ and $y = y_2$ corresponding, respectively, to $T = T_0$ and $T = T_2$ and to $N_i = N_{i0}$ and $N_i = N_{i2}$ (cf. Fig. 1) yields

$$q\int_0^{y_2}\frac{dy}{c\rho\kappa_e} = T_2 - T_0 + Q\int_{N_{i0}}^{N_{i2}}\frac{D_e}{(1-N_i)C\kappa_e}\,dN_i \ldots [5]$$

where $c\rho/n = C$ is the average molar isobaric heat capacity of the gas mixture. If no chemical recombination occurs, Equation [5] reduces to the following equation, in which the primes denote appropriate values subject to the initial assumption of constant T_2 and T_0

$$q'\int_0^{y_2'}\frac{dy}{c\rho\kappa_e} = T_2 - T_0 \ldots\ldots\ldots\ldots [5']$$

Since, as was stated in Section I, chemical heat sources have only a small effect on the Prandtl number (2), the boundary layer thickness y_2' may be taken to be the same as y_2. Furthermore, since the temperature limits over the boundary layer are the same, the integrals on the left-hand side of Equations [5] and [5'] may be equated within a reasonable approximation. The ratio of heat flux with and without recombination for identical boundary conditions in temperature is therefore

$$\frac{q}{q'} = 1 + \frac{Q}{T_2 - T_0}\int_{N_{i0}}^{N_{i2}}\frac{D+\epsilon}{(1-N_i)C(\kappa+\epsilon)}\,dN_i \ldots [6]$$

If more than one recombining species exists and if their combined mole fraction is low, Equation [6] can be easily generalized by summation of all the species i which react as follows

$$\frac{q}{q'} = 1 + \frac{1}{T_2 - T_0}\sum_{i=1}^{j} Q_i \int_{N_{i0}}^{N_{i2}}\frac{D_i+\epsilon}{(1-N_i)C(\kappa+\epsilon)}\,dN_i$$
$$\ldots\ldots [6']$$

where $i = 1, 2, 3, \ldots, j$. D_i is assumed to be an appropriate binary diffusion coefficient in the mixture.[5] In the three cases of Equation [6] that follow, it is assumed that this generalization can be made.

A Reaction in Boundary Layer Where $D_e = \kappa_e$

In this first case a simplified expression for the relative change in heat flux is obtained if either of the following two conditions is satisfied:

1 $D = \kappa$ with reaction occurring anywhere in the entire boundary layer.

2 Reaction occurring only in the hot portion of the intermediate layer where D and κ are much less than ϵ.

In either case, Equation [6] can be integrated directly to yield

$$\frac{q}{q'} = 1 + \frac{Q(N_{i2} - N_{i0})}{N_m\bar{C}(T_2 - T_0)} \ldots\ldots\ldots [7]$$

where N_m is the log-mean mole fraction of the remaining species. For the recombination of various labile species of low mole fraction

$$\frac{q}{q'} = 1 + \frac{\Sigma Q_i(N_{i2} - N_{i0})/(1 - N_{im})}{\bar{C}(T_2 - T_0)} \ldots\ldots [7']$$

where $\bar{C}(= \bar{c}M)$ is the average molar heat capacity of the gas mixture in the temperature range of recombination.[6] It is interesting that, in this case, the increase in heat transfer is no longer dependent on the transport properties but only on the ratio of recombination heat to sensible heat. Although N_{i2} and T_2 do not refer exactly to free-stream properties, such an identification can be made since, if the conditions of this first case hold, $D_e = \kappa_e$ beyond y_2, and integration of Equation [6] can be made into the free stream.

If the recombination rate is fast, chemical equilibrium is closely approached in the boundary layer. Since the recombination heats for most of the free radicals are usually high (some 50 kcal/mole and higher), it follows that recombination occurs largely in the high-temperature region of the intermediate layer where $\epsilon \gg \kappa$. This condition is probably approached for many reactions at high flame temperatures. If T_0 is 1000 K or so less than T_2, it is generally a good approximation to set N_{i0} equal to zero if chemical equilibrium is maintained, whereas N_{i2} and T_2 may be identified with equilibrium flame conditions. However, it is important to note that, regardless of whether or not chemical equilibrium is followed in the diffusion process, Equation [7] or [7'] is applicable. Hence the increase in heat transfer is independent of the manner or location of heat release except for the minor change of the molar heat capacity $\bar{C}$, which is averaged over the temperature range of chemical recombination.

Another simplification of Equation [7'] proves of practical utility for the case where N_{i0} is negligible. If T_2^0 denotes the calculated flame temperature for a combustion reaction where the products are assumed to be only those stable at ambient temperatures (i.e., CO_2, H_2O, H_2, CO, N_2, etc.), then the energy-balance equation yields

$$\overline{C^0}(T_2^0 - T_2) = \Sigma Q_i N_{i2} \ldots\ldots\ldots\ldots [8]$$

where $\overline{C^0}$ is the average molar heat capacity for the stable products between the temperatures T_2 and T_2^0. At the high temperatures where chemical dissociation is important, equipartition of energy is closely obeyed, and $\overline{C^0}$ and $\bar{C}$ differ only slightly, mainly as a result of a small composition difference and anharmonicity effects of the internal-energy states. Under the conditions that $\overline{C^0} = \bar{C}$, that $N_{im} \ll 1$ [so that $1/(1 - N_{im}) \simeq 1 + N_{im}$], and that the N_{i0} values are

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[5] The accurate treatment for multicomponent parallel diffusion through a stagnant layer is quite intractable. Sherwood (5) has given a solution for two parallel diffusing species which was obtained by Gilliland. Wilke (6) has indicated an approximate method of obtaining D_i in multicomponent mixture which may be suitable for this case.

[6] For recombination at or near chemical equilibrium, this average corresponds to a temperature close to T_2 because the degree of dissociation rises exponentially with temperature.

zero, Equation [8] may be incorporated into Equation [7'] to yield

$$\frac{q}{q'} = \frac{T_2^0 - T_0}{T_2 - T_0} + \frac{\Sigma Q_i N_{i2} N_{im}}{\bar{C}(T_2 - T_0)} \quad \text{.........} \quad [9]$$

It may be noted that the second term in Equation [9] is small with respect to the first and may be neglected for approximate purposes. In using Equation [9], T_2^0 is calculated very simply by writing the energy balance between combustibles and stable products for the single net stoichiometric equation neglecting dissociation.

B   Reaction Restricted to Surface

In the second case there are several types of reactions which are catalyzed by surfaces. Among such reactions are atomic and free-radical recombination, the endothermic dissociation of certain species such as ammonia, and certain chain reactions. The particular case of a reaction occurring solely on the surface can easily be treated in this scheme. Equations [1] through [6] are still valid in this second case, and if the restriction $D = \kappa$ quoted in the first case is satisfied, then Equation [7] can be employed. However, it is incorrect to employ Equation [7] if $\epsilon \gg D$ and κ and if $D \neq \kappa$, since the integral in Equation [6] extends over the entire boundary layer. In order to treat the general case of $D \neq \kappa$ (as occurs with H atoms, for instance), it is convenient to solve Equations [1] and [2] independently with $\Re = 0$ since it is assumed that no reaction occurs in the bulk phase. In this treatment the heat liberated at the wall by surface reaction is entirely absorbed there by an appropriate coolant flow which holds the temperature T_0 fixed. This condition implies that the occurrence of surface reaction has a negligible effect on the temperature contour in the boundary layer and that therefore the convective flow of heat q' is the same with and without surface reaction. The second integral of Equation [1] can be expressed as

$$T_2 - T_0 = q' \int_0^{y_2} \frac{dy}{c\rho(\kappa + \epsilon)} \quad \text{...........} \quad [10]$$

Denoting $\bar{c}$ and $\bar{\rho}$ as the mean values in the film, the resulting integral can be solved according to the assumption given by von Kármán (3). The dimensionless distance $y^* = (y/\nu)$ $(\sqrt{\tau/\rho})$ is employed in which it is assumed that $\epsilon = 0$ for $y^* < 5$ and $\epsilon/\nu = (y^*/5) - 1$ for $5 < y^* < 30$. Since $\nu/\kappa = Pr$, the result of the integration yields

$$q' = \lambda_0 \left(\frac{dT}{dy}\right)_0 = \frac{(T_2 - T_0)\bar{c}\bar{\rho}\sqrt{\tau/\bar{\rho}}}{5[Pr + \ln(1 + 5Pr)]} \quad \text{......} \quad [11]$$

A corresponding treatment can be made for solution of the conservation-of-mass equation (Equation [2]) with $\Re = 0$. Let F_i be the rate at which the diffusing species reacts per unit wall surface; then it is easily shown that

$$F_i = \frac{D n_0}{1 - N_i} \left(\frac{dN_i}{dy}\right)_0 = \frac{(N_{i2} - N_{i0})(\bar{n}/N_m)\sqrt{\tau/\bar{\rho}}}{5[Sc + \ln(1 + 5Sc)]} \quad \text{....} \quad [12]$$

The relative heat flux is therefore

$$\frac{q}{q'} = \frac{q' + QF_i}{q'} = 1 + \frac{Q(N_{i2} - N_{i0})}{N_m \bar{C}(T_2 - T_0)} \left[\frac{Pr + \ln(1 + 5Pr)}{Sc + \ln(1 + 5Sc)}\right] \quad \text{......} \quad [13]$$

Clearly, if $D = \kappa(Sc = Pr)$, Equation [13] reduces directly to Equation [7] as just stated. However, the surface recombination of H atoms, for which the $Pr > Sc$, causes a magnification of the heat release effect since the diffusion of H atoms carrying recombination heat exceeds that of the remaining molecules carrying thermal heat.

The remaining problem is the evaluation of N_{i0} in terms of the accommodation coefficient α, which is the fraction of molecular collisons with the wall leading to reaction. If Z_i

is the total number of surface collisons per unit time made by the diffusing species, the following equation[7] for F_i defines α

$$F_i = \alpha Z_i = \alpha n_0 N_{i0} \sqrt{\frac{RT_0}{2\pi M_i}} \quad \text{............} \quad [14]$$

Equations [12] and [14] may now be used to evaluate N_{i0} in terms of known quantities. Defining β as

$$\beta = \frac{\bar{n}\sqrt{\tau/\bar{\rho}}}{5n_0[Sc + \ln(1 + 5Sc)]\sqrt{\frac{RT_0}{2\pi M_i}}} \quad \text{.......} \quad [15]$$

the solution for N_{i0} is

$$N_{i0} = \frac{(\beta/\alpha)N_{i2}}{1 + (\beta/\alpha)} \quad \text{.............} \quad [16]$$

and Equation [13] reduces to

$$\frac{q}{q'} = 1 + \frac{QN_{i2}}{[1 + (\beta/\alpha)]N_m \bar{C}(T_2 - T_0)} \left[\frac{Pr + \ln(1 + 5Pr)}{Sc + \ln(1 + 5Sc)}\right] \quad \text{......} \quad [17]$$

For small values of β/α, $N_{i0} \to 0$, whereas for $\beta/\alpha \gg 1$, $N_{i0} \to N_{i2}$. As a typical example, consider a flame at 3000 K with a velocity of 100 fps at y_2 in a tube with the inner-wall temperature at 1000 K. With $Sc = 1$ and $M_i = 20$, the value of β is about 3.2×10^{-5}, and any value of $\alpha > 10^{-3}$ leads to $N_{i0} \simeq 0$; i.e., complete reaction at the surface.

C   Reaction Restricted to Laminar Layer

In the third case, if reaction occurs only in the laminar layer where $\epsilon = 0$, Equation [6] can be integrated between the limits $N_i = N_{i0}$ and $N_i = N_{i1}$ provided $D/\kappa = Pr/Sc$ is independent of N_i and T as predicted by kinetic theory for rigid spherical molecules. Under these restrictions, the result is

$$\frac{q}{q'} = 1 + \frac{QPr}{\bar{C}Sc \, N_m} \left(\frac{N_{i1} - N_{i0}}{T_1 - T_0}\right) \quad \text{.........} \quad [18]$$

For laminar flow where M_{i1} and T_1 are close to free stream properties, Equation [18] is useful for direct computation. For the general case including convective flow, an accurate solution can be obtained for N_1 and T_1 by the separate solutions of Equations [1] and [2] in the regions $y^* < 5$ and $5 < y^* < 30$. In the sublayer where $y^* < 5$, Equations [1] and [2] must be solved using an appropriate expression for $\Re = f(N_i, T)$. The unknown values T_1 and N_{i1} are then determined by matching with the solutions obtained from Equations [1] and [2] in the region $5 < y^* < 30$ where $\Re = 0$. However, it may be noted that, if $D = \kappa$, the solution reduces to Equation [7] even though reaction occurs only in the laminar layer. Such a situation may occur in hot flames containing easily dissociated species such as Br or I atoms in low concentration.

III   Specific Application

It may be of interest to examine the application of the equations derived to a specific flame system transferring heat to a cold wall. In most cases dealing with dissociated gases in high-temperature flames, the rates of chemical recombination are fast relative to the rate of transport through the boundary layer; thus, to a good approximation, the mole fraction of labile species at the wall may be considered negligible if the wall temperature is sufficiently low. The data represented in Tables 1 and 2 refer to the combustion products resulting from the reaction of liquid ammonia with gaseous oxygen at

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[7] Strictly speaking, n_0, N_{i0}, and T_0 should be evaluated one mean free path from the wall. However, for most practical cases, the magnitude of one mean free path is much less than y_1; consequently the simplification employed is valid.

Laminar Heat Transfer Over Blunt-Nosed Bodies at Hypersonic Flight Speeds

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This paper deals with two limiting cases of laminar heat transfer over blunt-nosed bodies at hypersonic flight speeds, or high stagnation temperatures: (a) thermodynamic equilibrium, in which the chemical reaction rates are regarded as "very fast" compared to the rates of diffusion across streamlines; (b) diffusion as rate-governing, in which the volume recombination rates within the boundary layer are "very slow" compared to diffusion across streamlines. In either case the gas density near the surface of a blunt-nosed body is much higher than the density just outside the boundary layer, and the velocity and stagnation enthalpy profiles are much less sensitive to pressure gradient than in the more familiar case of moderate temperature differences. In fact, in case (a), the nondimensionalized enthalpy gradient at the surface is represented very accurately by the "classical" zero pressure gradient value, and the surface heat-transfer rate distribution is obtained directly in terms of the surface pressure distribution. In order to illustrate the method, this solution is applied to the special cases of an unyawed hemisphere and an unyawed, blunt cone capped by a spherical segment.

In the opposite limiting case where diffusion is rate-controlling the diffusion equation for each species is reduced to the same form as the low-speed energy equation, except that the Prandtl number is replaced by the Schmidt number. The simplifications introduced in case (a) are also applicable here, and the expression for surface heat transfer rate is similar; the *maximum* value of the ratio between the rate of heat transfer by diffusion alone and by heat conduction alone in the case of thermodynamic equilibrium is given by: (Prandtl no./Schmidt no.)$^{2/3}$. When the diffusion coefficient is estimated by taking a reasonable value of atom-molecule collision cross section this ratio is 1.30. Additional theoretical and (especially) experimental studies are clearly required before these simple results are accepted.

Nomenclature

The subscript w refers to gas properties evaluated at the local surface temperature, while the subscript e denotes quantities at the outer edge of the boundary layer, and the subscript ∞ refers to ambient properties just ahead of the bow shock wave. The subscripts i and j represent the ith and jth chemical species. The subscript o generally denotes quantities at the forward stagnation point. A prime denotes differentiation with respect to η.

a = speed of sound
C_i = concentration by weight of the ith species
C_v, C_p = specific heats at constant volume and constant pressure, respectively, per unit mass ($\bar{C}_p = \Sigma C_i C_{p_i}$)
$C(\eta)$ = $\rho\mu/\rho_e\mu_e$, Chapman-Rubesin function
D = diameter
D_{ij} = coefficient of diffusion of species "i" into species "j"

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f = u/u_e
g = h_s/h_{s_e}
h = static enthalpy
$h_i^{\,o}$ = standard enthalpy of formation $\}$ per unit mass
h_s = total enthalpy
k = coefficient of thermal conductivity
M = Mach number u/a
$\mathfrak{m}$ = molecular weight
Nu = Nusselt number, hD/k, where $q_w = h/C_{p_e}(h_{s_e} - h_w)$
p = static pressure
Le = Lewis number, $\rho D_{12}\bar{C}_p/k$
Sc = Schmidt number $\mu/\rho D_{12}$
p_o' = stagnation pressure behind normal shock wave
Pr = Prandtl number, $\bar{C}_p\mu/k$
$\dot{q}$ = local heat transfer rate, per unit area, per unit time
r_0 = radius of cross section of body of revolution
r = distance between centers of two molecules or atoms
R_0 = nose radius
$\mathfrak{R}_0$ = universal gas constant
$\mathfrak{R}$ = $\mathfrak{R}_0/\mathfrak{m}$
s = distance along body surface, measured from forward stagnation point
$\bar{s}$ = transformed coordinate along body surface (Eq. [7])
T = absolute temperature
T^* = kT/ϵ, where k is the Boltzmann const = 1.380×10^{-16} erg/°K
u, v = components of velocity parallel and normal to surface
w_i = net rate of production of ith species, gm/cm^3/sec
x = distance along body axis, measured from forward stagnation point
y = distance normal to surface
z_i = nondimensional concentration
β = velocity gradient parameter, $(2\bar{s}/u_e)(du_e/d\bar{s})$
$\bar{\beta}$ = $(2\bar{s}/M_e)(dM_e/d\bar{s})$
γ = ratio of specific heats, C_p/C_v
δ = boundary layer thickness
δ^* = boundary layer displacement thickness, $\int_0^\infty [1 - (\rho/\rho_e) u/u_e] dy$
ϵ = maximum attraction energy, erg
η = nondimensional coordinate normal to body surface (Eq. [7])
θ = angle between flight direction and radius vector from center of curvature of nose
θ_b = angle of inclination of local surface element with respect to flight direction
θ_c = cone half-angle
μ = absolute viscosity
ν = kinematic viscosity, μ/ρ
ρ = gas density
σ = zero-energy distance, or collision diameter
$\varphi(r)$ = interaction potential for intermolecular force field
ψ = stream function
$\Omega_{ij}^{(l,\,m)*}(T^*)$ and $\Omega_i^{(l,\,m)*}(T^*)$, collision integrals
ω_e = $\mu_e/\mathfrak{R}_e T_e$

1 Introduction

AT HYPERSONIC flight speeds all bodies must be blunt-nosed to some extent in order to reduce the heat transfer rates to manageable proportions and to allow for internal heat conduction. Since the maximum local heat transfer rate in most cases will occur in the vicinity of the forward stag-

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nation point, the fluid-mechanical problems associated with this region are of considerable practical, as well as theoretical, interest. Because of the strong falling pressure gradient and large heat transfer to the surface, the boundary layer in this region is expected to be laminar and therefore amenable to analytic treatment.

At hypersonic flight speeds the detached bow shock wave just ahead of a blunt-nosed body converts most of the kinetic energy associated with the flight velocity into thermal and chemical energy. At the "satellite escape speed" of 26,400 ft/sec, for example, this energy amounts to about 14,000 Btu per lb of air, which is sufficient to dissociate almost all of the nitrogen and oxygen molecules. The mechanism of heat transfer in the boundary layer under these conditions may differ markedly from the purely molecular heat conduction process of conventional fluid mechanics. For this reason it is desirable to examine certain limiting cases in order to become familiar with the relative importance of the various possible physical processes, and to provide a basis for comparison with experiment. Two important limiting cases are:

1 Complete thermodynamic equilibrium, in which the gas properties and concentrations of atoms, molecules, and ions are identical with their equilibrium values appropriate to the local temperature at every point of the flow. Here the chemical and electronic reaction rates are regarded as "very fast" in comparison with the rates of convection along streamlines, or the rates of diffusion across streamlines.

2 Diffusion as rate-governing for heat transfer. In this opposite limiting case the volume recombination rates within the boundary layer are "very slow" compared to diffusion across streamlines, because of the relatively low temperatures near the surface. Heat transfer from the boundary layer to the body surface is accomplished partly by ordinary heat conduction, and partly by the heat released by catalytic recombination of those atoms that manage to penetrate the screen of molecules diffusing away from the surface.[2]

The actual physical situation is expected to lie somewhere between these two limiting cases, at least at flight altitudes low enough so that the gas can still be regarded as a "continuum," and heat transfer to the body surface is still of major importance.

Even in these idealized limiting cases the laminar boundary layer problem is not simple, unless certain special approximations are introduced. Cohen and Reshotko (1)[3] extended Thwaites' approximate momentum-integral method based on the "similar solutions" to compressible flows with constant specific heats. Also, S. Lal (2) developed straight-forward techniques for applying the momentum and energy integral methods to such flows. In a recent paper Hayes[4] shows how the Stewartson-Illingworth transformation, which reduces the problem to an equivalent low-speed flow, is generalized (at least approximately) to the case of an arbitrary imperfect gas. He also discusses the various methods and approximations available for solving the reduced problem. By following Hayes' approach, these methods could now be modified to include the case of temperature-dependent specific heats.

Fortunately, at hypersonic flight speeds, as we shall see in Section 2.2, the problem is simplified considerably by recognizing that the gas density near the surface of a blunt-nosed body is much higher than the density just outside the boundary layer. The ratio ρ_w/ρ_e is large, i. e. 10–20, not only because $T_w/T_e \ll 1$, but also because the molecular weight of the gas near the surface is roughly twice the average molecular weight just outside the boundary layer. Since the pressure gradient in the direction parallel to the surface depends on the "external" density, ρ_e, and is uniform across the boundary layer,

the velocity profile near the surface is much less sensitive to the pressure gradient than in the more familiar case of moderate temperature difference across the boundary layer. The stagnation enthalpy distribution is even less sensitive. In fact, for $\rho_w/\rho_e \gg 1$, we will show that an excellent first approximation is obtained by ignoring the direct effect of pressure gradient entirely in comparison with the influence of the local pressure on the gas density.[5] The velocity profile is then identical with the Blasius distribution in properly transformed coordinates, and the heat transfer rate distribution over the surface of a blunt body is obtained directly in terms of the surface pressure distribution.

Since the nondimensionalized enthalpy gradient at the surface is independent of surface temperature in this approximation, the method is applicable to nonisothermal surfaces, so long as $\rho_w/\rho_e \gg 1$. The calculations are particularly simple when the modified Newtonian flow approximation is employed, and they can be carried out for a blunt-nosed cylindrical body as well as for a body of revolution. In Section 3 the heat transfer rate solution of the present paper is applied to the special cases of an unyawed hemisphere and an unyawed blunt cone capped by a spherical segment.

The methods developed here are also applicable to the second limiting case where diffusion is rate-controlling. The diffusion equation for each species is reduced to the same form as the low-speed energy equation, except that the Schmidt number $\mu/\rho D_{12}$ replaces the Prandtl number (Section 4). For $\rho_w/\rho_e \gg 1$, the maximum value of the ratio between the heat energy that would be transferred to the surface by diffusion alone, and by heat conduction alone is given by $(\rho D_{12}\bar{C}_p/k)_w{}^{2/3}$. When the diffusion coefficient D_{12} is estimated by taking reasonable values of atom-molecule collision cross section, this ratio is found to be 1.30. Additional theoretical and (especially) experimental studies of this problem are clearly required before these simple results can be accepted.

2 Laminar Boundary Layer Equations for an Atom-Molecule Mixture of Perfect Gases

2.1 Relative Magnitude of Heat Transfer by Diffusion and by Heat Conduction: The Parameter $(\rho D_{12}\bar{C}_p)/k$

Before examining the laminar boundary layer equations including diffusion in some detail, it is helpful to have some estimate of the relative magnitude of the rate of heat transfer by diffusion and by heat conduction. Because of the close similarity between the transport properties and atomic weights of oxygen and nitrogen, it is sufficient to treat the gas in the boundary layer as a binary mixture of atoms and molecules, so far as diffusion is concerned, in which case one deals with a single diffusion coefficient, D_{12}.[6] Also the concentration of nitric oxide is generally low enough so that its contribution to the energy transfer is negligible. For the same reason one is justified in ignoring the presence of the ions and electrons in a discussion of boundary layer heat transfer at flight Mach numbers less than about 25.

With these simplifications the heat transfer rate is given by

$$q = -k\frac{\partial T}{\partial y} + \sum \rho C_i V_i h_i = -\left[k\frac{\partial T}{\partial y} + \rho D_{12}\sum h_i \frac{\partial C_i}{\partial y} \right]$$

where V_i, the diffusion velocity, is equal to $-(D_{12}/C_i)(\partial C_i/\partial y)$ according to Fick's law, and

$$h_i = \int_0^T C_{pi} dT + h_i{}^0$$

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[2] The present paper does not consider the problem of radiative heat transfer from the hot gases outside the boundary layer to the surface.

[3] Numbers in parentheses indicate References at end of paper.

[4] Private communication.

[5] This approximation was first employed by the present author in treating the hypersonic boundary-layer shock-wave interaction problem (3), but it appears to be generally useful.

[6] The author is indebted to Dr. S. S. Penner of the California Institute of Technology for this suggestion.

where h_i^0 is the heat of formation of the ith species. Now, $dh = \bar{C}_p dT + \sum h_i\, dC_i$, where $\bar{C}_p = \sum C_i C_{pi}$, so that

$$\dot{q} = -\frac{k}{\bar{C}_p}\left[\left(\frac{\partial h}{\partial y} - \sum h_i \frac{\partial C_i}{\partial y}\right) + \frac{\rho D_{12}\bar{C}_p}{k}\sum h_i \frac{\partial C_i}{\partial y}\right]$$

In the limiting case of local thermodynamic equilibrium the atom concentration vanishes some distance away from the surface, provided the surface temperature is below the dissociation limit at the local pressure, p_s. At the surface

$$\left(\frac{\partial C_i}{\partial y}\right) = 0, \quad \text{and} \quad \left(\frac{\partial h}{\partial y}\right)_w = \left(\frac{\partial h_s}{\partial y}\right)_w \sim \frac{h_{s_e} - h_w}{\delta}$$

so that

$$(\dot{q}_w)_{\text{equil}} \sim \frac{1}{\delta}\frac{k_w}{\bar{C}_{p_w}}(h_{s_e} - h_w)$$

In the opposite limiting case where the volume recombination rates in the gas phase are slow compared to the diffusion rates

$$\left(\frac{\partial C_i}{\partial y}\right)_w \sim \frac{C_{i_e}}{\delta}$$

for the atoms, on the assumption that the atom concentration is zero at the surface, while for the molecules

$$\left(\frac{\partial C_i}{\partial y}\right)_w \sim \frac{C_{i_e} - C_{i_w}}{\delta}$$

Therefore

$$\dot{q}_w \sim \frac{1}{\delta}\frac{k_w}{\bar{C}_{p_w}}(h_{s_e} - h_w)\left[\underbrace{\left\{1 - \frac{\sum h_{i_w}(C_{i_e} - C_{i_w})}{(h_{s_e} - h_w)}\right\}}_{\text{conduction}} + \underbrace{\left(\frac{\rho D_{12}\bar{C}_p}{k}\right)_w \frac{\sum h_{i_w}(C_{i_e} - C_{i_w})}{(h_{s_e} - h_w)}}_{\text{diffusion}}\right]$$

In other words, the relative magnitude of the heat transferred by the two processes is governed by the parameters

$$\left(\frac{\rho D_{12}\bar{C}_p}{k}\right)_w \quad \text{and} \quad \frac{\sum h_{i_w}(C_{i_e} - C_{i_w})}{(h_{s_e} - h_w)}$$

At sufficiently high flight speeds this last parameter is nearly unity, and diffusion is responsible for practically all of the heat transfer. In this case, the surface heat transfer rate at a given flight speed (given h_{s_e}) is larger than the value for thermodynamic equilibrium roughly by the factor $(\rho D_{12}\bar{C}_p/k)_w$. When $(\rho D_{12}\bar{C}_p/k)_w = 1$, *the surface heat transfer rate is independent of the mechanism of heat transfer at any flight speed, and is exactly equal to the value for thermodynamic equilibrium.*[7]

According to the kinetic theory of gases the quantity $\rho D_{ii}\bar{C}_p/k$ is of the order of unity, where D_{ii} is the coefficient of self-diffusion. In fact, for nitrogen molecules $\mu/\rho D_{ii} = 0.74$ and $c_p\mu/k = 0.71$ at $T = 273\,°\mathrm{K}$ and $p = 1$ atm, so that $(\rho D_{ii}c_p)/k = 0.96$.[8] But here we require the value of D_{12}, which involves atom-molecule collisions rather than collisions between molecules; an estimate of this coefficient is made in Section 4.2. One expects D_{12} to be larger than D_{ii} if only because lighter and faster atomic particles are involved; therefore $(\rho D_{12}\bar{C}_p/k)_w > 1$. For this reason, it is desirable to obtain a somewhat closer estimate of the relative magnitude of the two modes of heat transfer by examining the boundary layer equations. Our procedure is to deal first with the case of thermodynamic equilibrium in some detail; many of the techniques developed here are immediately applicable to the case where diffusion is rate-controlling for heat transfer (Section 4).

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[7] A similar conclusion was reached by R. Bromberg using a different approach (private communication).
[8] Table 1.2–3, page 16, Ref. (4).

In order to bring out the main features of the problem as simply as possible, consider the steady flow over an unyawed, blunt-nosed body of revolution (or a cylindrical body). The basic equations of laminar boundary layer flow are as follows

Continuity Equation

$$\frac{\partial}{\partial s}\left(\rho u r_0{}^k\right) + \frac{\partial}{\partial y}\left(\rho v r_0{}^k\right) = 0 \quad\ldots\ldots\ldots\ldots [1]$$

where $k = 0$ for a planar body and $k = 1$ for a body of revolution.[9]

Momentum Equation

$$\rho\left(u\frac{\partial u}{\partial s} + v\frac{\partial u}{\partial y}\right) = -\frac{dp_e}{ds} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)\quad\ldots\ldots [2]$$

Energy Equation

$$\rho\left(u\frac{\partial h_s}{\partial_s} + v\frac{\partial h_s}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\mu}{Pr}\frac{\partial h_s}{\partial y}\right) +$$
$$\frac{\partial}{\partial y}\left\{\mu\left(1 - \frac{1}{Pr}\right)\frac{\partial}{\partial y}\frac{u^2}{2}\right\} + \frac{\partial}{\partial y}\left\{\rho D_{12}\left(1 - \frac{1}{Le}\right)\sum h_i \frac{\partial}{\partial y}C_i\right\}$$
$$\ldots\ldots [3]$$

Continuity Equation for Each Species

$$\rho u\frac{\partial C_i}{\partial x} + \rho v\frac{\partial C_i}{\partial y} - \frac{\partial}{\partial y}\left(\rho D_{12}\frac{\partial C_i}{\partial y}\right) = w_i\quad\ldots\ldots [4]^{10}$$

Equation of State

$$p = \rho\frac{R_0}{\mathfrak{m}}T, \quad \text{where} \quad \mathfrak{m} = \left[\sum\frac{C_i}{\mathfrak{m}_i}\right]^{-1}\quad\ldots\ldots [5]$$

Since $h_s = \sum_i C_i h_i + u^2/2$, it includes the chemical enthalpy $\sum C_i h_i^0$.

Clearly there are two cases in which the energy equation takes the familiar form:

1. Lewis number $= \rho D_{12}\bar{C}_p/k = 1$.
2. $C_i = $ constant across the boundary layer.

In the first case, as shown previously, the heat transfer rate is given by the expression

$$\dot{q} = -\frac{k}{\bar{C}_p}\frac{\partial h}{\partial y}$$

independently of the mechanism of heat transfer. In the second case, *either* the gas temperatures are low enough so that no dissociation has occurred, *or* no chemical reactions take place either in the gas phase or at the surface; i.e., the gas composition is "frozen."

Apart from these general remarks not much progress can be made until one finds the "similar solutions," or one-parameter family of velocity and total enthalpy profiles. In this respect the situation is not very different from the low-speed boundary layer problem.

2.2 Similarity Considerations for Thermodynamic Equilibrium: The Case of the Highly Cooled Surface

In the limiting case of thermodynamic equilibrium the concentration of each species is uniquely related to the local pressure and temperature, and the net rate of production of each species w_i is supposed to be large enough to balance the

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[9] For Reynolds numbers based on nose radius that are not "too low," $\delta/r_0 \ll 1$ and the continuity equation takes this reduced form. Also, $\mathcal{K}\delta$ and $\delta^2\partial\mathcal{K}/\partial s$ are assumed to be "small," where $\mathcal{K}$ is body surface curvature. The boundary layer and shock wave are distinct when $\delta_0/\delta_s \sim (\rho_s/\rho_\infty)/\sqrt{u_\infty R_0/\nu_\infty} < 0.10$ (say), or roughly when $(u_\infty R_0/\nu_\infty) > 10^4$. (Here δ_s is the separation distance between the bow shock and the forward stagnation point.)
[10] These equations are readily derived from the general equations given by Hirschfelder, et al. (Ref. 4). Thermal diffusion is considered to be negligible compared with mass diffusion.

convection and diffusion terms. In other words, Equation [4] is superfluous in this case. Also, tables or graphs of static enthalpy and molecular weight vs. temperature for air are either available or can be constructed by methods similar to those described in (5b).[10a] By means of the relation $\rho/\rho_e = (T_e/T)(\mathfrak{M}/\mathfrak{M}_e)$ the density ratio is then uniquely determined by the enthalpy ratio h/h_e, for each value of h_e and the pressure p_e across the boundary layer.

Our procedure is to assume that "similarity" exists and then to determine the conditions (if any) under which this assumption is in fact justified. For this purpose the momentum and energy equations are reduced to the more convenient equivalent "low-speed" form by utilizing Levy's transformation (6), which he introduced for the special case of constant specific heats.[11] In order to simplify the discussion the Lewis number is taken to be unity; this restriction is not an essential one, and in no way affects the validity of the general arguments.

Let

$$\left. \begin{aligned} \eta &= \frac{\rho_e u_e}{(2\bar{s})^{1/2}} \int_0^y r_0{}^k \rho/\rho_e \, dy \\ \bar{s} &= \int_0^s \rho_e \, \mu_e u_e r_0{}^{2k} \, ds \end{aligned} \right\} \quad \ldots\ldots\ldots\ldots [6]$$

The transformation from s, y coordinates to $\bar{s}, \eta$ coordinates is carried out by means of the relations

$$\left. \begin{aligned} \frac{\partial}{\partial y} &= \frac{\rho u_e r_0{}^k}{(2\bar{s})^{1/2}} \frac{\partial}{\partial \eta} \\ \frac{\partial}{\partial s} &= \rho_e \mu_e u_e r_0{}^{2k} \left[\frac{\partial}{\partial \bar{s}} + \frac{\partial}{\partial \eta} \frac{\partial \eta}{\partial \bar{s}} \right] \end{aligned} \right\} \quad \ldots\ldots\ldots\ldots [7]$$

The over-all continuity equation is automatically satisfied by introducing the stream function ψ, which is defined by the usual relations

$$\rho u r_0{}^k = \frac{\partial \psi}{\partial y} \ldots\ldots\ldots\ldots\ldots\ldots [8a]$$

and

$$\rho v r_0{}^k = -\frac{\partial \psi}{\partial s} \ldots\ldots\ldots\ldots\ldots\ldots [8b]$$

Let $\psi(s, \eta) = (2\bar{s})^{1/2} f(\eta)$; then by Equations [7] and [8a], $u/u_e = f'(\eta)$, where the prime denotes differentiation with respect to η. Also define $h_s/h_{s_e} = g(\eta)$. The momentum equation then takes the form

$$(Cf'')' + ff'' + \frac{2\bar{s}}{u_e} \frac{du_e}{d\bar{s}} \left[\frac{\rho_e}{\rho} - (f')^2 \right] = 0 \ldots\ldots [9]$$

and the energy equation is

$$\left(\frac{C}{Pr} g' \right)' + fg' + \frac{u_e{}^2}{2h_{s_e}} \left[2C \left(1 - \frac{1}{Pr} \right) f'f'' \right]' = 0 \ldots [10]$$

where

$$C = \frac{\rho \mu}{\rho_e \mu_e}$$

The boundary conditions are as follows:

$f(0) = f'(0) = 0$, and $f'(\eta) \to 1$ as $\eta \to \infty$;
$g(0) = g_w(s)$, or $g'(0) = 0$, while $g(\eta) \to 1$ as $\eta \to \infty$.

For an isoenergetic flow outside the boundary layer the conditions for similarity are:

(a) $(2\bar{s}/u_e)(du_e/d\bar{s})[\rho_e/\rho - (f')^2] = F(\eta)$, or a const.
(b) $C = C(\eta)$, or a const.

[10a] Because the gas is treated as a binary mixture of atoms and molecules, the mass ratio of oxygen (atoms + molecules) to nitrogen is constant along every streamline and equal to the stoichiometric ratio for the standard atmosphere.

[11] Here we combine Levy's transformation with the Mangler transformation for bodies of revolution.

(c) Either (a) $Pr = 1$ or (b) $Pr = Pr(\eta)$, or a const, while either $u_e{}^2/2h_{s_e} \ll 1$ (low local Mach number) or $u_e{}^2 \to 2h_{s_e}$ (high local Mach number).
(d) $g_w(s) = $ const.

Except for the special case of a uniform external flow, all of these conditions for similarity are never satisfied simultaneously at hypersonic flight Mach numbers, even if the surface temperature is uniform. The density ratio ρ_e/ρ, the Prandtl number, and the ratio $\rho\mu/\rho_e\mu_e$ are functions not only of h/h_e, but also of h_e itself, and of p_e through its effect on the composition of the gas. Of course it is always possible to find a "locally similar" solution in the neighborhood of the forward stagnation point that is "exact" so long as the boundary layer and bow shock are distinct. The boundary layer thickness h_e and p_e are all very nearly constant along the surface in this region, and the velocity and enthalpy ratios are functions of a single variable of the form Y/R_0 $\sqrt{u_e R_0/\nu_e}$, where $Y = \int_0^y (\rho/\rho_e) \, dy$. The question naturally arises as to whether one can obtain approximate solutions extending over most of the body surface. Such solutions can be developed in at least two cases of interest: (a) slowly varying external flow properties; (b) highly cooled surface, or $g_w \ll 1$.

In the first case the approximation of "local similarity" is employed, in which the velocity and enthalpy profiles at any station are supposed to be identical with the "similar profiles" corresponding to constant values of

$$\beta = \left(\frac{2\bar{s}}{u_e} \frac{du_e}{d\bar{s}} \right)_{local} \quad \text{and} \quad (u_e{}^2/2h_{s_e})_{local}$$

all along the surface, and to the local relations between ρ_e/ρ, C, Pr, and the enthalpy ratio h/h_e. This approximation is utilized by Stine and Wanlass (7) (for example) in the special case of constant specific heats and ordinary stagnation temperatures, where $\rho_e/\rho = h/h_e$ and C and Pr are both practically constant.

In the second case, where the surface temperature is much lower than T_e, or $\rho_e/\rho_w \ll 1$, the influence of the pressure gradient term in Equation [9] should be small according to the qualitative physical argument in Section 1, and the problem is simplified considerably. This argument is supported in two ways: (a) rough estimates; (b) examination of known solutions for the special case of constant specific heats. At hypersonic flight speeds

$$\frac{\rho_e}{\rho} \cong \frac{h_s}{h_{s_e}} = g(\eta)$$

when the difference in molecular weight across the boundary layer is taken into account. Now

$$g \cong g_w + (1 - g_w) u/u_e \cong u/u_e = f'(\eta)$$

when $g_w \ll 1$.[12] Therefore, the term in brackets in Equation [9] is represented approximately by $[u/u_e - (u/u_e)^2]$, which is zero both at the surface and at the outer edge of the boundary layer, and has a maximum value of $1/4$ within the boundary layer. Thus, at most, the effect of the pressure gradient of the velocity profile is equivalent to the case of an isothermal low-speed flow with an "effective" value of $\beta = (2\bar{s}/u_e)(du_e/d\bar{s})$ equal to one fourth of the actual value of this parameter.[13]

[12] This relation is "exact" when the pressure gradient is zero, and either $Pr = 1$, or $(u_e{}^2/2h_{s_e}) \ll 1$.
[13] The first-order effect of the pressure gradient term on the skin friction, for example, is given by

$$\frac{(2\bar{s}/u_e)(du_e/d\bar{s}) \int_0^\infty e^{-\int_0^\eta f d\eta} \left\{ \int_0^\eta e^{\int_0^\eta f d\eta} [\quad] d\eta \right\} d\eta}{\int_0^\infty e^{-\int_0^\eta f d\eta} d\eta}$$

where $[\quad] = (\rho_e/\rho) - (f')^2 \cong u/u_e - (u/u_e)^2$ and $f(\eta)$ is the Blasius function.

At the forward stagnation point the parameter $(2\bar{s}/u_e)$ $(du_e/d\bar{s})$ is equal to $1/2$ for a body of revolution and unity for a cylinder. The maximum "effective" values of β when $\rho_e/\rho_w \ll 1$ are $1/8$ and $1/4$, respectively, and the influence of the pressure gradient on the velocity profile is much reduced, compared to the case of an insulated surface. But the effect of the pressure gradient on the heat transfer rate is even smaller, because the change in $g'(0)$ from its zero pressure gradient value depends upon an integral of the change in the velocity profile across the boundary layer. In fact

$$g'(0) \cong \frac{1 - g_w}{\displaystyle\int_0^\infty e^{-Pr \int_0^\eta f \, d\eta} \, d\eta}$$

These conclusions are borne out by the results of the numerical calculations of Cohen and Reshotko (8) for the "similar solutions," with the particular assumption of constant specific heats and $Pr = 1$, $C = 1$. In that case

$$\rho_e/\rho = g + (\gamma - 1)/2 \, M_e^2 \, [g - (f')^2]$$

and Equations [9] and [10] become

$$f''' + ff'' + \bar{\beta} \, [g - (f')^2] = 0$$

and

$$g'' + fg' = 0$$

with

$$\bar{\beta} = (2\bar{s}/M_e) \, (dM_e/d\bar{s}) = \text{const}$$

The surface shear stress and heat transfer rate functions f_w'' and $g_w'/1 - g_w = -s_w'/s_w$ as calculated by Cohen and Reshotko (8) are shown as functions of $\bar{\beta}$ in Figs. 1a and 1b, for the cases $g(0) = h_w/h_{se} = 0$, 0.20, and 1.0.[14] Clearly the approximation that the direct influence of pressure gradient is negligibly small for $\rho_e/\rho_w \ll 1$ is highly accurate for the surface heat transfer rate, and somewhat less accurate for skin-friction.[15] Similar conclusions can be drawn from the numerical results obtained by Levy (Fig. 10 of Ref. 6), and by Lal (Fig. 5 of Ref. 2).

With regard to the function $C(\eta)$ appearing in Equations [9] and [10], extrapolation of existing data on the viscosity of air by means of Sutherland's formula shows that at high enthalpy levels (30–100 times ordinary enthalpy), $\rho\mu \cong \rho_e\mu_e$ over a large portion of the boundary layer (see Ref. 5c, for example). Therefore so far as the heat transfer rate is concerned, it is sufficient to take $C = 1$, especially when we recall that C enters the problem as a square root. Also, in the present state of our knowledge, it seems reasonable to regard the Prandtl number as a constant, equal to some average value $\bar{Pr}$ across the boundary layer (Section 4.2).

With regard to the energy equation (Sp. [10]), the solution can always be chosen so that the inhomogeneous last term does not contribute to the enthalpy gradient at the surface; in fact, this term determines the "recovery" enthalpy, or surface enthalpy for no heat transfer, as shown by Chapman and Rubesin (9). Of course this term vanishes at a stagnation point, in which case the recovery enthalpy is identical with h_{se}. In any event this term is numerically small on a blunt-nosed body at hypersonic flight speeds, where $(u_e^2/2h_{se})_{max} \cong 1/2$ and $\bar{Pr} \cong 0.65$–1.00.

These considerations suggest that the nondimensionalized enthalpy gradient $g'(0)$ at the surface of a blunt-nosed body at hypersonic flight speeds is represented quite accurately by the zero pressure gradient value of $0.47 \, \bar{Pr}^{1/3}$, independently of surface temperature, local Mach number M_e, or pressure gradient, and is therefore applicable to a very general class of surfaces, whether isothermal or not, so long as $\rho_w/\rho_e \gg 1$.[16]

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[14] $g = 1 + s$.
[15] From Figs. 1a and 1b it appears that the "effective" value of $\bar{\beta}$ is more nearly one sixth of the actual value.
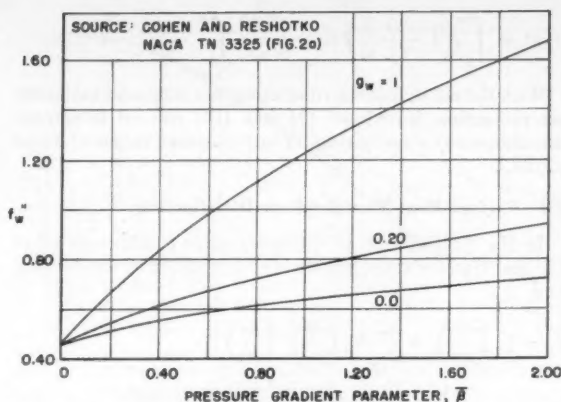
Fig. 1(a) Effect of pressure gradient and ratio of surface temperature to stagnation temperature on surface shear stress function, $Pr = 1$, $C = 1$
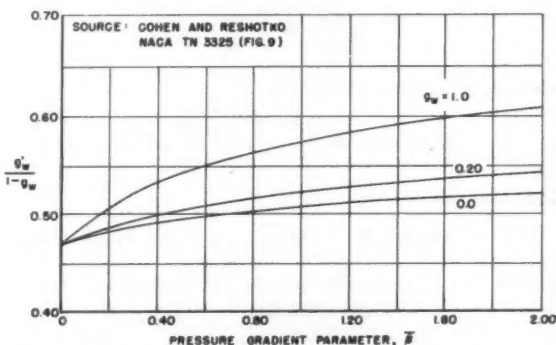
Fig. 1(b) Effect of pressure gradient and ratio of surface temperature to stagnation temperature on surface heat transfer rate function, $Pr = 1$, $C = 1$

If other quantities are required which depend more sensitively on the distribution of $\rho\mu$ and ρ across the boundary layer, they can be calculated by means of a straightforward iteration process. In the first step the stagnation enthalpy distribution $g^{(0)}(\eta)$ is obtained from Equation [10] by repeated quadratures, with $f^{(0)}(\eta)$ given by the Blasius function, $u_e^2/2h_{se}$ equal to its local value, $C = 1$ and $Pr = \bar{Pr}$. (Here the local similarity approximation is employed.) By substituting the corresponding density profile $\rho_e^{(0)}/\rho$ and the function

$$C^{(0)}(\eta) = \frac{\rho^{(0)} \, \mu^{(0)}}{\rho_e \, \mu_e}$$

into the momentum equation (Equation [9]), the next approximation $f^{(1)}(\eta)$ is calculated by numerical integration for the local value of

$$\beta = \frac{2\bar{s}}{u_e} \frac{du_e}{d\bar{s}}$$

and the process is repeated, if necessary. Finally, the actual distance normal to the surface is computed by means of the quadrature

$$y = \frac{(2\bar{s})^{1/2}}{r_0^k \rho_e u_e} \int_0^\eta \rho_e/\rho \, d\eta$$

and the displacement thickness (for example) is given by the expression

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[16] This approximation is clearly *not* applicable near boundary layer separation.

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$$\delta^* = \int_0^\infty \left(1 - \frac{\rho}{\rho_e}\frac{u}{u_e}\right)dy = \frac{(2\bar{s})^{1/2}}{r_0^k \rho_e u_e}\int_0^\infty (\rho/\rho_e - u/u_e)\,d\eta$$

With the aid of modern computing machines and tabulated air properties, Equations [9] and [10] can be integrated simultaneously step-by-step for any constant values of β and $u_e^2/2h_{se}$.

2.3 Surface Heat Transfer Rate Distribution

In the limiting case of thermodynamic equilibrium, all of the heat transfer at the surface is accomplished by conduction, and

$$\dot{q}_w = k_w \left(\frac{\partial T}{\partial y}\right)_w = \frac{k_w}{C_{p_w}} h_{se} \left(\frac{dg}{d\eta}\right)_w \left(\frac{\partial \eta}{\partial y}\right)_w$$
$$\cong \frac{k_w}{\mu_w C_{p_w}} \frac{\mu_w \rho_w u_e r_0^k}{(2\bar{s})^{1/2}} h_{se} (0.50\,\bar{P}r^{1/3}) \dots [11]$$

for $\rho_w/\rho_e \gg 1$, taking round numbers. Now $\rho_w\mu_w = \rho_e\mu_e$, to be consistent, and

$$\frac{\rho_e\mu_e}{(\rho_e\mu_e)_0} = \frac{p}{p_0'}\frac{\omega_e}{\omega_{e_0}}$$

where $\omega_e = \mu_e/\mathcal{R}_e T_e$. Therefore

$$2\bar{s} = 2(\rho_e\mu_e)_0 u_\infty \int_0^s p/p_0' \frac{u_e}{u_\infty}\frac{\omega_e}{\omega_{e_0}} r_0^{2k}\,ds$$

and

$$\dot{q}_w \cong 0.50\,(\bar{P}r)^{-2/3}\sqrt{(\rho_e\mu_e)_0}\sqrt{u_\infty}\,h_{se}\cdot F(s)\dots[12]$$

where

$$F(s) = \frac{(1/\sqrt{2})\,(p/p_0')\,(\omega_e/\omega_{e_0})\,(u_e/u_\infty)\,r_0^k}{\left[\int_0^s (p/p_0')\,(u_e/u_\infty)\,(\omega_e/\omega_{e_0})\,r_0^{2k}\,ds\right]^{1/2}}$$

Here $k = 0$ for a planar body and $k = 1$ for a body of revolution, and $\bar{P}r$ is some average Prandtl number [see Section (4.2)]. In particular at the forward stagnation point

$$\dot{q}_w)_0 \cong 0.50\,2^{k/2}(\bar{P}r)^{-1/3}\sqrt{(\rho_e\mu_e)_0}\sqrt{u_\infty}\,h_{se}$$
$$\frac{\sqrt{[(R/u_\infty)\,(du_e/ds)]_{s=0}}}{\sqrt{R_0}}\dots[12a]$$

Some years ago Squire (10) worked out the solution for the convective heat transfer near the forward stagnation point of a cylinder on the assumption of constant fluid properties (small temperature differences). More recently, Sibulkin (11) obtained an analogous solution near the forward stagnation point of a blunt body of revolution. Their results are expressed in the form

$$Nu_D = \frac{hD}{k_2} = 0.570\,(Pr_2)^{0.4}\,(\sqrt{2})^k \sqrt{\left(\frac{D}{u_2}\frac{du_e}{ds}\right)_0}\sqrt{\frac{u_2 D}{\nu_2}}$$

where $\dot{q}_0 = h(T_{se} - T_w)$, and the subscript 2 denotes quantities evaluated in the gas just behind the normal shock, following Sibulkin's suggestion.[17] For purposes of comparison, define h in our case by the relation $(\dot{q}_w)_0 = (h/C_{p_e})\,(h_{se} - h_w)$; then our result is expressed as

$$Nu_{D.e} = \frac{hD}{k_e} = (0.50)\,2^{k/2}(\bar{P}r)^{1/3}\sqrt{\left(\frac{D}{u_2}\frac{du_e}{ds}\right)_0}\sqrt{\frac{u_2 D}{\nu_e}}$$

As expected, the present result has the same form as the

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[17] Cohen and Reshotko (12) have applied their similar solutions for a compressible fluid in the special case of constant specific heats and $C = Pr = 1$ to the forward stagnation point, with similar results. They also investigated the effect of fluid injection.

"low-speed," constant-property solutions, except that now the value of $[(D/u_2)\cdot(du_e/ds)]_0$ is obtained from the Newtonian flow approximation at hypersonic speeds, and not from the irrelevant low-speed potential flow over a cylinder or a sphere.

The expression for $Nu_{D.e}$ at the forward stagnation point is of the same form as the result obtained by Beckwith (13), who utilized the momentum and energy integral method, including an approximate density-enthalpy relation for dissociated air in thermodynamic equilibrium.

Also, R. Mark (14) worked out the differential equations for the similar solutions in the neighborhood of the forward stagnation point, and integrated these equations numerically, utilizing the equilibrium thermal properties of dissociated air given in (5c). (Lewis number = 1.) When his result for the stagnation point heat transfer rate is expressed in the same form as our Equation [12a], then his quantity λ [(Eq. (14) and Fig. 5 of Ref. 14)], multiplied by $\rho_w\mu_w/\rho_e\mu_e$, should be compared with the numerical value $0.50\sqrt{2} = 0.70$ (more precisely with $0.47\sqrt{2} = 0.67$). For surface temperatures of 3000 and 5000 R, and over a range of values of stagnation enthalpy from 50–120 times the ordinary static enthalpy of air, the value of $\lambda(\rho_w\mu_w/\rho_e\mu_e)$ differs from 0.67 by 5 per cent at most. For $T_w = 1000$ R, the value of this quantity is about 10 per cent lower than 0.67. This result tends to justify the simplifying approximation $\rho\mu = \rho_e\mu_e$ utilized in this paper.

As discussed in Section 4 of (15), a careful examination of the experimental data on pressure distributions over hemisphere cylinders and blunt cones with spherical noses shows that the modified Newtonian law in the form $C_p/C_{p_{max}} = \sin^2\theta_b$ is confirmed for $M_\infty > 2$ (approximately). Therefore, in Equation [12], $p/p_0' = \sin^2\theta_b + (p_\infty/p_0')\cos^2\theta_b$, and

$$\frac{u_e^2}{u_\infty^2} \cong \left(1 + \frac{2}{\gamma_\infty - 1}\frac{1}{M_\infty^2}\right)\left[1 - \left(\frac{p}{p_0'}\right)^{(\bar{\gamma}-1)/\gamma}\right]$$

where $\bar{\gamma}$ is the mean ratio of specific heats behind the bow shock wave. (At high temperatures, $\bar{\gamma} \cong 1.10$–1.20.) Near the forward stagnation point $\sin^2\theta_b = 1 - \sin^2\theta$, so that

$$\frac{u_e}{u_\infty} \cong \sqrt{\frac{\bar{\gamma}-1}{\bar{\gamma}}}\sqrt{1 + \frac{2}{\gamma_\infty - 1}\frac{1}{M_\infty^2}}\sqrt{1 - \frac{1}{\gamma_\infty M_\infty^2}}\sin\theta + \dots$$

where $p_\infty/p_0' \cong 1/\gamma_\infty M_\infty^2$ and

$$\left(\frac{1}{u_\infty}\frac{du_e}{d\theta}\right)_{\theta=0} \cong \sqrt{\frac{\bar{\gamma}-1}{\bar{\gamma}}}\sqrt{1 + \frac{2}{\gamma_\infty - 1}\frac{1}{M_\infty^2}}\sqrt{1 - \frac{1}{\gamma_\infty M_\infty^2}}^{18}$$

The stagnation point heat transfer rate is then

$$(\dot{q}_w)_0 \cong \frac{0.50\,2^{k/2}(\bar{P}r)^{-1/3}\sqrt{(\rho_e\mu_e)_0}\sqrt{u_\infty}\,h_{se}\cdot G(M_\infty;\bar{\gamma},\gamma_\infty)}{\sqrt{R_0}}$$
$$\dots[13]$$

where

$$G(M_\infty;\bar{\gamma},\gamma_\infty) = \left(\frac{\bar{\gamma}-1}{\bar{\gamma}}\right)^{1/4}\left(1 + \frac{2}{\gamma_\infty - 1}\cdot\frac{1}{M_\infty^2}\right)^{1/4}$$
$$\left(1 - \frac{1}{\gamma_\infty M_\infty^2}\right)^{1/4}$$

In flight at hypersonic speeds $h_{se} = u_\infty^2/2$; in the wind tunnel h_{se} is usually the reservoir stagnation temperature; in the shock tube $h_{se} = u_s^2$ for sufficiently strong shocks, where u_s is the velocity of the shock wave in the straight section.

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[18] This expression agrees with the experimental data (16) obtained on a hemisphere cylinder to within 3 per cent in the range $1.7 \leq M_\infty \leq 5$, with $\bar{\gamma} = \gamma_\infty = 1.40$.

3 Surface Heat Transfer Rate Distribution over Unyawed Hemisphere and Blunt Cone

According to the modified Newtonian flow approximation, for the particular case of an unyawed body of revolution with a constant radius of curvature in the meridian plane, $p/p_0' = \cos^2\theta + (1/\gamma_\infty M_\infty^2)\sin^2\theta$, where θ is the angle between the radius vector and the flight direction. By retaining the second term in this last expression the heat transfer rate distribution obtained is quite accurate even for Mach numbers as low as 2.0, and joins smoothly with the distribution over an afterbody of arbitrary shape.

The velocity at the outer edge of the boundary layer can always be obtained from the isentropic flow relation with an effective γ. But the predicted velocity over a hemisphere varies very nearly linearly with θ up to $\theta = 80$ deg, and the experimental data for air in the range $1.97 < M_\infty < 5.8$ show a remarkable linearity (15, 16). In the present calculations, thereore, we take $u_e/u_\infty = (1/u_\infty)(du_e/d\theta)_{\theta=0}\theta$. On the conical skirt

$$u_e/u_\infty = \left(\frac{1}{u_\infty}\frac{du_e}{d\theta}\right)_{\theta=0}\cdot\left(\frac{\pi}{2}-\theta_c\right)$$

Because of the slow variation of the quantity $\omega_e = \mu_e/R_e T_e$ with enthalpy h_e at high enthalpy levels it is sufficient to take $\omega_e \cong \omega_{eo}$. With this simplification the ratio of the local surface heat transfer rate to the heat transfer rate at the forward stagnation point is given by

$$\frac{\dot{q}_w}{(\dot{q}_{w_o})} = \frac{(1/2)(p/p_0')(u_e/u_\infty)r_0}{\left[\int_0^s (p/p_0')(u_e/u_\infty)r_0^2\,ds\right]^{1/2}}\times$$
$$\frac{\sqrt{R_0}}{\sqrt{(1/u_\infty)(du_e/d\theta)_0}}\quad\ldots\ldots[14]$$

Case I   Hemisphere:
For this body $ds = R_0 d\theta$, $r_0 = R_0\sin\theta$; therefore

$$\frac{\dot{q}_w}{\dot{q}_{w_o}} = \frac{1/2\theta\sin\theta[\cos^2\theta+(1/\gamma_\infty M_\infty^2)\sin^2\theta]}{\left\{\int_0^\theta \theta\sin^2\theta[\cos^2\theta+(1/\gamma_\infty M_\infty^2)\sin^2\theta]\,d\theta\right\}^{1/2}}$$

or

$$\frac{\dot{q}_w}{(\dot{q}_w)_o} = \frac{2\theta\sin\theta\{[1-(1/\gamma_\infty M_\infty^2)]\cos^2\theta+(1/\gamma_\infty M_\infty^2)\}}{[D(\theta)]^{1/2}}\ ..[15]$$

where

$$D(\theta) = \left(1-\frac{1}{\gamma_\infty M_\infty^2}\right)\left(\theta^2-\frac{\theta\sin4\theta}{2}+\frac{1-\cos4\theta}{8}\right)+$$
$$\frac{4}{\gamma_\infty M_\infty^2}\left(\theta^2-\theta\sin2\theta+\frac{1-\cos2\theta}{2}\right)\ \ldots\ldots[15a]$$

Near the forward stagnation point

$$\frac{\dot{q}_w}{(\dot{q}_w)_o} = 1-\left(0.722-\frac{0.667}{\gamma_\infty M_\infty^2}\right)\theta^2+\ldots$$

In Fig. 2 the ratio $\dot{q}_w/(\dot{q}_w)_o$ is plotted as a function of θ for $M_\infty = 2, 3, 5, 10,$ and ∞, with $\gamma_\infty = 1.40$. The quantity $\sqrt{p/p_0'}$ for $M_\infty = 2$ is also shown in Fig. 3 for comparison. Evidently the heat transfer rate drops off somewhat faster than $\sqrt{p/p_0'} \sim \sqrt{\rho_e\mu_e}$. By dividing $\dot{q}_w/(\dot{q}_w)_0$ by $\sqrt{p/p_0'}$ we obtain a quantity proportional to the local heat transfer parameter

$$Nu/\sqrt{Re} = \frac{hx/k_e}{\sqrt{\rho_e u_e x/\mu_e}}$$

employed by Korobkin (16) and by Stine and Wanlass (7). For $M_\infty = 2$, the values of $Nu/\sqrt{Re}$ calculated by the present method for $T_w/T_e \ll 1$ are about 12 to 15 per cent lower
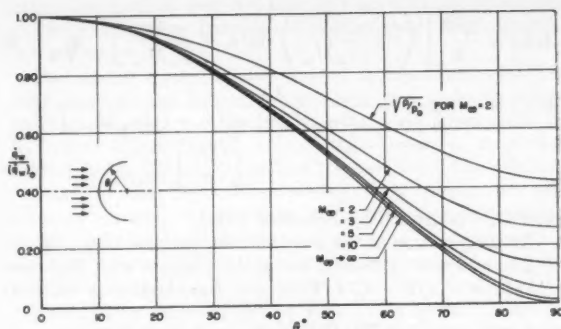
Fig. 2 Laminar heat transfer rate distribution over isothermal hemispherical nose, with $(T_w/T_e)\ll 1$
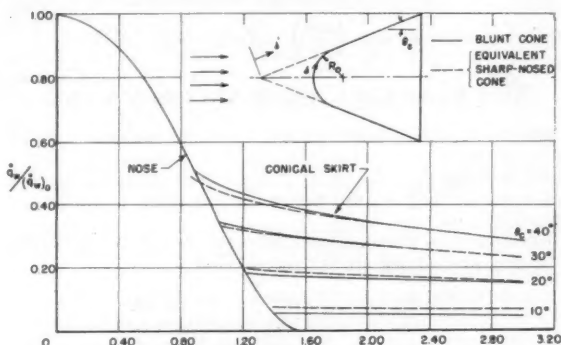
Fig. 3 Laminar heat transfer rate distribution over blunt cone, with $(T_w/T_e)\ll 1$ $M_\infty \sin\theta_c \gg 1$

than the theoretical values obtained by Stine and Wanlass for $T_w/T_e \cong 1$ (solid curve in Fig. 4b of Ref. 7). This difference is almost entirely accounted for by the fact that the nondimensionalized enthalpy gradient at the surface for $T_w/T_e \cong 1$ is 12 to 15 per cent higher than the Blasius value. (See Fig. 1b, $\bar\beta = 1/2$; $g_w = 1$, for example.) Unfortunately the scatter in the experimental data (7, 16) so far obtained is too great to permit any detailed comparisons to be made with the theoretical results.

Except for the portion of the hemisphere aft of $\theta = 60$ deg (approx.) the heat transfer rate distribution is close to the "frozen" hypersonic distribution when $M_\infty > 2$. For $M_\infty > 5$ the heat transfer rate is close to its hypersonic values over practically the entire hemisphere. Actually the static pressure is somewhat higher over the region near $\theta = 90°$ than predicted by the Newtonian approximation, so the predicted values of $\dot{q}_w/(\dot{q}_w)_0$ there should be regarded mainly as indicative of relative magnitudes.

Case II   Blunt Cone Capped by a Spherical Segment:
For this body the surface heat transfer rate distribution over the "nose" is of course identical with that found for the forward region of the hemisphere [$\theta \leqq \pi/2 - \theta_c$, where θ_c is the cone half angle (see Fig. 3)]. On the conical skirt all physical quantities are regarded as constants, and $r_0 = s'\sin\theta_c$, where s', the distance along the surface measured from the "virtual tip," is related to the actual distance s by the expression (Fig. 3)

$$\frac{s'}{R_0} = \cot\theta_c+\left[\frac{s}{R_0}-(\pi/2-\theta_c)\right]$$

By substituting the expressions for p/p_0', u_e/u_∞, and r_0 into Equation [14], one obtains

$$\frac{\dot{q}_w}{(\dot{q}_w)_0} = A(\theta_c)\frac{s'/R_0}{[B(\theta_c)+(s'/R_0)^3]^{1/2}}$$

for $s'/R_0 \geq \cot\theta_c$, where

$$A(\theta_c) = \frac{\sqrt{3}}{2}\left[\left(1 - \frac{1}{\gamma_\infty M_\infty^2}\right)\sin^2\theta_c + \frac{1}{\gamma_\infty M_\infty^2}\right]^{1/2}\sqrt{\frac{\pi}{2} - \theta_c}$$

$$B(\theta_c) = \frac{^{3/16}}{\sin^3\theta_c\left\{[1 - (1/\gamma_\infty M_\infty^2)]\sin^2\theta_c + (1/\gamma_\infty M_\infty^2)\right\}} \times \left[\frac{D(\theta)}{\theta}\right]_{\theta=(\pi/2)-\theta_c} - \cot^3\theta_c$$

and $D(\theta)$ is defined by Equation [15a].

At the junction of the nose and the conical skirt, $s'/R_0 = \cot\theta_c$, and this expression for $\dot{q}_w/(\dot{q}_w)_0$ agrees with Equation [15] for $\theta = \pi/2 - \theta_c$. "Far" from the junction ($s'/R_0 \gg 1$)

$$\dot{q}_w \rightarrow \sqrt{3}\,(0.35)\,(\overline{Pr})^{-1/2}\,\sqrt{\rho_c\mu_c}\,\sqrt{u_c}\,h_{s_e}(1/\sqrt{s'})$$

which is exactly the value of the surface heat transfer rate on a sharp-nosed cone with an "equivalent velocity"

$$u_c = \left(\frac{d\mu_c}{d\theta}\right)_{\theta=0}(\pi/2 - \theta_c)$$

When $M_\infty \sin\theta_c \gg 1$, $A(\theta_c) \cong \sqrt{3/2}\,\sin\theta_c \cdot \sqrt{(\pi/2) - \theta_c}$

and

$$B(\theta_c) \cong {}^{3/16}\frac{1}{\sin^4\theta_c}\left[\frac{D(\theta)}{\theta}\right]_{\theta=\pi/2-\theta_c} - \cot^3\theta_c$$

The numerical values of these functions for $\theta = 10$ deg, 20 deg, 30 deg, and 40 deg are tabulated below.

θ_c°	$A(\theta_c)$	$B(\theta_c)$
10	0.181	172.0
20	0.326	+4.2
30	0.443	−0.20
40	0.520	−0.26

Evidently for $\theta_c = 30$ deg and 40 deg the surface heat transfer rate distribution over the conical skirt is practically identical with the distribution for the "equivalent sharp-nosed cone." For slender cones, however, the heat transfer rate is smaller than the value for a sharp-nosed cone for a distance of several nose radii aft of the junction of nose and skirt. These conclusions are illustrated quantitatively in Fig. 3.

4 Diffusion as Rate Controlling for Heat Transfer

4.1 Surface Heat Transfer Rate

When the volume recombination rates in the boundary layer can be disregarded in comparison to the rate of diffusion across streamlines, the diffusion equation for each species takes the following form (Equation [4], Section 2.1, with $w_i = 0$):

$$\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial y} = \frac{\partial}{\partial y}\left(\rho\,D_{12}\frac{\partial C_i}{\partial y}\right)\ldots\ldots[16]$$

By introducing the transformations of Equation [7], and defining $z_i(\eta) = C_i/C_{i_e}$ for the atoms, and

$$z_i(\eta) = \frac{C_{i_w} - C_i}{C_{i_w} - C_{i_e}}$$

for the molecules, Equation [16] is reduced to an equation for $z_i(\eta)$ of the same form as the energy equation (Equation [10]) with the term proportional to $u_e^2/2h_{s_e}$ suppressed, except that the Prandtl number is now replaced by the Schmidt number, or the parameter $\mu/\rho D_{12}$. By the arguments employed in Section 2.2, it follows that when $\rho_e/\rho_w \ll 1$, $z_i'(0) \cong 0.50\,(\mu/\rho D_{12})_w^{-1/2}$, and the rate of heat transfer to the surface by diffusion is

$$(\dot{q}_w)_{\text{diff}} = (\rho D_{12})_w \left(\frac{\partial\eta}{\partial y}\right)_w\left[\sum_i h_{i_e}(C_{i_e} - C_{i_w})\right]\cdot 0.50\left(\frac{\mu}{\rho D_{12}}\right)_w^{1/2}$$
$$\ldots\ldots[17]$$

By comparing this expression with Equation [11], one obtains

$$\frac{(\dot{q}_w)_{\text{diff}}}{(\dot{q}_w)_{\text{equil}}} = \left(\frac{\rho D_{12}\bar{C}_p}{k}\right)_w^{1/2}\frac{\sum_i h_{i_e}(C_{i_e} - C_{i_w})}{h_{s_e}}$$

where $(\dot{q}_w)_{\text{equil}}$ is the rate at which heat is transferred to the surface by conduction in the limiting case of thermodynamic equilibrium. *In other words, at a given flight speed the maximum difference in surface heat transfer rate between two limiting cases is measured by the factor $(\rho D_{12}\bar{C}_p/k)_w^{1/2}$.*

For the particular case in which the surface temperature is high enough so that all the C_{p_i}'s are nearly the same (while $\rho_w/\rho_e \gg 1$), the enthalpy transferred by conduction can be treated separately from the full enthalpy transfer. The full static enthalpy equation for the laminar boundary layer is as follows:

$$\sum_i\frac{\partial}{\partial x}(\rho u C_i h_i) + \sum_i\frac{\partial}{\partial y}\rho(v + V_i)C_i h_i =$$
$$\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + u\frac{dp_e}{dx}\ldots\ldots[18]$$

By utilizing Equation [16] and the fact that $\sum\rho c_i V_i = 0$, Equation [18] is reduced to the familiar form

$$\rho u\,\bar{C}_p\frac{\partial T}{\partial x} + \rho v\,\bar{C}_p\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + u\frac{dp_e}{dx}$$

where

$$\bar{C}_p = \sum C_i C_{p_i} \quad \text{and} \quad h_T = \int_0^T\bar{C}_p\,dT$$

Without going into details it is clear that in this case the surface heat transfer rate by conduction alone is given by

$$(\dot{q}_w)_{\text{cond.}} \cong 0.50\,(Pr_w)^{1/3}\left(\frac{\partial\eta}{\partial y}\right)_w k_w[h_{s_e} - \sum h_{i_w}(c_{i_e} - c_{i_w})]$$

and

$$\frac{(\dot{q}_w)_{\text{cond.}} + (\dot{q}_w)_{\text{diff}}}{(\dot{q}_w)_{\text{equil}}} \cong \left\{\left[1 - \frac{\sum h_{i_w}(C_{i_e} - C_{i_w})}{h_{s_e}}\right] + \left(\frac{\rho c_p D_{12}}{k}\right)_w^{1/2}\frac{\sum h_{i_w}(C_{i_e} - C_{i_w})}{h_{s_e}}\right\}\ldots\ldots[19]$$

In this particular case the differential equation for the partial enthalpy

$$\left(\frac{h_T + u^2/2}{h_{T_e} + u_e^2/2}\right) = g_T(\eta)$$

is identical with Equation [10], and the temperature distribution is obtained by a method similar to that described in Section 2.2. The concentration profiles are obtained by numerically integrating the equation

$$z_i'' + \left(\frac{\mu}{\rho D_{12}}\right)_w f z_i' = 0 \quad \text{with} \quad f = f^{(0)} = f_B(\eta) \text{ and } C^{(0)} = 1^{19}$$

and the boundary conditions $z_i(0) = 0$, $z_i(\eta) \rightarrow 1$ as $\eta \rightarrow \infty$. Once $z_i(\eta)$ is known, the density profile is obtained from the relation

$$\frac{\rho_e}{\rho} = \frac{\mathfrak{m}_e}{\mathfrak{m}}\frac{T}{T_e}$$

where $\mathfrak{m}(\eta) = [\sum(C_i(\eta)/\mathfrak{m}_i)]^{-1}$. Of course iteration schemes similar to the method described at the end of Section 2.2 can be developed in this case also.

When the c_{p_i}'s are not all nearly the same the problem is somewhat more complicated and the details will not be given

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[19] $f_B(\eta)$ is the Blasius function.

here. In any event the ratio of the total surface heat transfer rates in the two limiting cases considered will lie between the value given in Equation [19] and $(\rho D_{12}\bar{C}_p/k)_w^{2/3}$.

4.2 Estimate of the Diffusion Coefficient D_{12} and the Parameters $\mu/\rho D_{12}$ and $(\rho D_{12}\ \bar{C}_p/k)$ According to the Kinetic Theory of Gases

In recent years several different groups of investigators have been remarkably successful in predicting the transport properties of dilute, nonpolar gases[20] and gas mixtures with the aid of approximate interaction potentials for the intermolecular force field. Hirschfelder and co-workers have employed the two-parameter Lennard-Jones potential

$$\varphi(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

where r is the distance between the centers of the two molecules, ϵ is the depth of the potential "well," or the maximum energy of attraction, and σ is the zero-energy distance, or separation distance at the point of closest approach for low-energy collisions ($\pi\sigma^2$ is the collision cross section). The inverse sixth-power term takes into account the long-range attractive forces, while the inverse twelfth-power term approximates the strong, short-range repulsive forces that are called into play when the electron clouds surrounding the two molecules begin to interpenetrate.[21]

For any potential function of the type $\varphi(r) = \epsilon f(r/\sigma)$ the first approximation to the diffusion coefficient of a binary mixture is given by

$$(D_{12})_1 = 0.002628\sqrt{\frac{T^3(\mathfrak{m}_1 + \mathfrak{m}_2)}{2\mathfrak{m}_1\mathfrak{m}_2}}\ \frac{1}{p\sigma_{12}{}^2\Omega_{12}{}^{(1,1)*}(T_{12}{}^*)}\quad \ldots [20]$$

in cm^2/sec, where p is the pressure in atmospheres; T is the absolute temperature in °K; $\mathfrak{m}_1$, $\mathfrak{m}_2$ are the molecular weights of species 1 and 2, respectively; $T_{12}{}^* = kT/\epsilon_{12}$, where k is the Boltzmann constant; σ_{12}, ϵ_{12}/k are the interaction parameters for collisions between species 1 and 2, in Å and °K, respectively, and $\Omega_{12}{}^{(1,1)*}(T_{12}{}^*)$ is the collision integral for mass transport; this integral is a measure of the departure from the rigid-sphere model, for which $\Omega_{12}{}^{(1,1)*} = 1.0$.[22] The higher approximations to D_{12} are given by $(D_{12})_k = (D_{12})_1\ f_{D_{12}}{}^{(k)}$, where $f_{D_{12}}{}^{(k)}$ depends on the mixture ratio, but for most binary mixtures $1.00 \leq f_{D_{12}}{}^{(2)} \leq 1.03$ for the Lennard-Jones potential. In other words, D_{12} is virtually independent of the mixture ratio.

For low energies [$T_{12}{}^* = 0(1)$] the collision integral for the Lennard-Jones potential is larger than unity, while for high energies $\Omega_{12}{}^{(1,1)*} < 1$. The Lennard-Jones molecule has a larger cross section than the rigid-sphere model at low energies, because of the importance of the attractive forces, and a smaller cross section at high energies to allow for the interpenetration against the repulsive forces. The function $\Omega^{(1,1)*}(T^*)$ is tabulated for $0.30 \leq T^* \leq 400$ in Table 1-M, pp. 1126–1127 of the Appendix to (4).

In principle, measurements of the diffusion coefficient provide the best source of information about interactions between unlike molecules, but such measurements are usually very limited. In general the interaction parameters σ_{12} and ϵ_{12} are approximated by the combining "rules" $\sigma_{12} = (\sigma_1 + \sigma_2)/2$ and $\epsilon_{12} = \sqrt{\epsilon_1\ \epsilon_2}$, where σ_1, σ_2, ϵ_1, and ϵ_2 are determined from measurements of the viscosity of the pure components of the binary mixture. As we shall see, the value of σ_{12} for atom-molecule diffusion determined in this manner is too low, be-

cause this method does not take into account the important physical differences between atom-atom and atom-molecule interactions.

Fortunately a considerable amount of theoretical work has been done on the hydrogen atom-hydrogen molecule interaction, and also on the molecule-molecule interaction in hydrogen. Experimental data on the viscosity of molecular hydrogen provide a check on the theoretical results obtained for the second case. By studying these results for the simplest element, we are able to make a reasonable estimate of the difference between the known collision cross section for molecule-molecule interactions in air, and the required collision cross-section for atom-molecule interactions.

Even for the "simple" hydrogen molecule the calculation of the short-range repulsive forces in the H_2-H_2 interaction is so complicated that reasonable approximations must be introduced (see Ref. 4, pp. 1083–1092). Four possible orientations of the molecules are considered, as follows

When the spatial average of the potential over all four orientations is calculated, it is found that a Lennard-Jones function fits this potential very closely, provided $\sigma = 2.928$ Å and $\epsilon = 5.107 \times 10^{-15}$ erg. It is remarkable that these values agree to within 1 per cent with the values of σ and ϵ obtained from viscosity data on molecular hydrogen.

On the basis of rough physical considerations, one might expect that the "collision diameter," $\sigma_{\text{H-H}_2}$, would be smaller than $\sigma_{\text{H}_2\text{-H}_2}$ by a distance of the order of one half the equilibrium separation r_e of the atoms in H_2, or about 0.37 Å. Margenau (17) considered the atom-molecule configurations
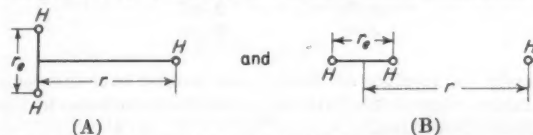
He calculated the repulsive part of the potential in each case by a first-order perturbation method, while the attractive portion of the potential is approximated quite accurately by the sum of an inverse sixth-power and an inverse eighth-power term. For configuration A, $\sigma_{\text{H-H}_2} = 2.65$ Å and $\epsilon_{\text{H-H}_2} = 5 \times 10^{-5}$ erg; for B, $\sigma_{\text{H-H}_2} = 3.03$ Å and $\epsilon_{\text{H-H}_2} = 3.5 \times 10^{-15}$ erg. Configuration A appears to be much more probable, and the potential is surprisingly similar to the potential for configurations (b) and (c) in the H_2-H_2 interaction, at least in the range 2.50 Å $\leq r \leq 4.0$ Å. Thus the collision diameter for atom-molecule interaction in hydrogen is about 0.28 Å smaller than $\sigma_{\text{H}_2\text{-H}_2}$ or $\sigma_{\text{H}_2\text{-H}_2} - \sigma_{\text{H-H}_2} \cong 3/4\ (r_e/2)$, while the maximum attraction energy in the two cases is virtually identical.

Now the best experimental value of the distance between the two nitrogen atoms in the N_2 molecule is about 1.1 Å. On the basis of the results of the study of the H-H_2 and H_2-H_2 interactions, it seems quite reasonable to suppose that the collision diameter for atom-molecule collisions in air is smaller than the value $\sigma = 3.69$ Å[23] for molecule-molecule interaction by about $3/4(r_e/2)$, or about 0.4 Å. In other words, $\sigma_{12} \cong 3.69$ Å $- 0.4$ Å $= 3.3$ Å. Also $\epsilon_{12}/k = \epsilon_{\text{air}}/k = 84$ K, at least for 300 K $< T <$ 1900 K. The diffusion coefficient D_{12} is to be obtained from Equation [20] with these values of σ_{12} and ϵ_{12}/k.[24]

Suppose we consider first the viscosity and conductivity of the gas at the surface, where the concentration of atomic

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[20] Nonpolar molecules are those for which the potential function does not depend on the orientations of the two interacting molecules, but only on the distance between centers.
[21] An excellent account of the whole modern development of the molecular theory of liquids and gases is given in (4).
[22] For the rigid-sphere model $\varphi(r) = 0$ for $r > \sigma$, and $\varphi(r) = \infty$ for $r < \sigma$. Å $\equiv$ 1 Ångström unit $= 10^{-8}$ cm.

[23] Table I-A, Appendix, Ref. (4).
[24] In Ref. (18) the value of σ_{12} is obtained from the rough approximation $\sigma_{12} = (\sigma_1 + \sigma_2)/2$, and σ_1 is taken as $1/2\ \sigma_2$, i.e., $\sigma_{12} = 3/4\ \sigma_2 \cong 2.77$Å. This value seems to be far too low. (Here the subscript "1" denotes the atomic and the subscript "2" denotes the molecular species.)

species is zero. Experimental data on the viscosity of molecular air are reproduced to within 1 per cent in the temperature range $300\ K < T < 1500\ K$, and to within 3 per cent in the temperature range $1500\ K < T < 1900\ K$, by the following expression based on the Lennard-Jones potential:

$$\mu \times 10^7 = 266.93\ \frac{\sqrt{m_2 T}}{\sigma_2{}^2 \Omega_2{}^{(2,\ 2)*}(T_2{}^*)} \quad \text{in gm/cm-sec} \ldots [21]$$

where $\Omega_2{}^{(2,\ 2)*}(T_2{}^*)$ is the collision integral for momentum transport. By Equations [20] and [21] the Schmidt number for atom-molecule diffusion in air is

$$\left(\frac{\mu}{\rho D_{12}}\right)_w \cong 0.833\ \frac{\sigma_{12}{}^2}{\sigma_2{}^2}\ \frac{\Omega_{12}{}^{(1,\ 1)*}(T_{12}{}^*)}{\Omega_2{}^{(2,\ 2)*}(T_2{}^*)}\ \frac{1}{\sqrt{\frac{1}{2}\left[1 + m_2/m_1\right]}}$$

where 0.833 is the value of the Schmidt number for the idealized case of self-diffusion in a monatomic gas according to the rigid-sphere model. But $\epsilon_{12}/k \cong \epsilon_2/k = 84\ K$, so that $T_{12}{}^* = T_2{}^*$, and the ratio of the collision integrals $= 1/A^*(T^*)$, where $A^*(T^*)$ is a very slowly varying function of temperature given in Table 1-N, Appendix, Ref. (4). Also $m_2 = 2m_1$ and $\sigma_{12} = 3.3$ Å, while $\sigma_2 = 3.69$ Å, so that

$$\left(\frac{\mu}{\rho D_{12}}\right)_w \cong \frac{0.543}{A^*(T^*)}$$

A few calculated values of the Schmidt number are tabulated below:

T, °K	$(\mu/\rho D_{12})_w$
252	0.495
420	0.492
840	0.489
1680	0.485
3360	0.482

Clearly the variation in the Schmidt number over this temperature range is less than the probable inaccuracies in the estimated value itself.

For a monatomic gas or a mixture of such gases the thermal conductivity is predicted by the kinetic theory with the same accuracy as the viscosity coefficient. But for polyatomic gases the question of the exchange of energy between the translational and internal degrees of freedom enters the problem. For dry air the vibrational relaxation times for N_2 and O_2 are known to be long at temperatures below 1000 K; thus, one might expect the thermal conductivity to behave as if only the translation and rotational degrees of freedom were involved. In other words the Prandtl number should be practically constant at its value for room temperature up to about $T = 1000\ K$. According to the NBS-NACA Tables (Table 2.44, Ref. (5a)) the Prandtl number for dry air based on the best fit to experimental values of μ and k, and spectroscopic data on $C_p{}^{(0)}$, varies from 0.708 at 300 K to a minimum of 0.680 for $500\ K \leq T \leq 600\ K$, and then rises again to 0.702 at $T = 1000\ K$. This variation in Prandtl number is less than the indicated scatter in the experimental data on thermal conductivity in Fig. 1, p. 5 of Table 2.42, Ref. (5a). Of course at somewhat higher temperatures the vibrational mode is much more readily excited, and the Prandtl number should increase gradually to the undissociated Eucken value of 0.783. Unfortunately no experimental data exist at temperatures above 1000 K, and the Eucken formula is known to give values of Prandtl number at $T = 300\ K$ that are about 5 per cent too high. For the present, we take $\overline{Pr} = 0.71$.

Finally the parameter $(\rho D_{12}\bar{C}_p/k)_w$ governing the relative importance of heat transfer by atom-molecule diffusion and by thermal conduction is given by

$$\left(\frac{\rho D_{12}\bar{C}_p}{k}\right)_w = \frac{(\bar{C}_p\mu/k)_w}{(\mu/\rho D_{12})_w} \cong \frac{0.71}{0.49} \cong 1.45$$

and the maximum value of the ratio of the rate at which heat is transferred by diffusion and by heat conduction is $(1.45)^{3/2}$, or about 1.30.

In Sections 2.2 and 4.1 it was pointed out that the effect of the variation in Schmidt number, Prandtl number, and the quantity $C(\eta) = \rho\mu/\rho_w\mu_w$ across the boundary layer can be taken into account by means of straightforward iteration schemes, or by simultaneous integration of the equations of motion with the aid of computing machines. For the case of thermodynamic equilibrium the properties of air are given in (5c). However, the calculations of air properties in this report are based on values of atom-atom collision diameter $\cong 1.5$ Å, which are far too large. According to the quantum-mechanical treatment of the N-N interaction by Kopineck (19), the equilibrium separation distance is 1.109 Å, which agrees very well with the experimental value of 1.095 Å, while the collision diameter $\sigma_1 = 0.80$ Å. Also $\epsilon_1/k = 1.14 \times 10^5$. On the other hand the values of atom-molecule collision diameter utilized in (5c) and (18), namely $\sigma_{12} = (\sigma_1 + \sigma_2)/2 \cong 2.8$ Å, seem to be too small, if the estimate of the present paper is correct. While the effects on Prandtl number of the difference in these two sets of estimated values of σ_1 and σ_{12} is small, the effect on the Schmidt number and $C(\eta)$ may be significant.

For the second limiting case in which the volume recombination rates are regarded as negligible in comparison with the diffusion rates, the viscosity of the gas considered as a binary atom-molecule mixture is calculated from Equation 8.2-30, page 533 of (4), while the Prandtl number is obtained by the method described by Hansen (18), except that the correct values of σ_1, σ_{12}, and σ_2 should be employed. The diffusion coefficient D_{12} is obtained from Equation [20] of the present paper. The following values are recommended for the interaction parameters.

$$\sigma_1 = 0.80 \text{ Å}, \qquad \epsilon_1/k = 1.14 \times 10^5\ K$$
$$\sigma_{12} = 3.3 \text{ Å}, \qquad \epsilon_{12}/k = \epsilon_2/k = 84\ K$$
$$\sigma_2 = 3.69 \text{ Å}$$

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Effect of Chemical Reactions in the Boundary Layer on Convective Heat Transfer

stoichiometric mixture ratio and a total pressure of 20 atmospheres. Inasmuch as recombination is presumed to occur at equilibrium, the stoichiometry of the various reactions shown in Table 2 was arbitrarily chosen to yield stable end products, i.e., N_2 and H_2O. The total percentage increase in heat transfer calculated in Table 2 is 20.7 per cent. The use of the approximate simple formula given by Equation [9] employing a "nondissociated" flame temperature T_2^0 of 3555 K shows a heat transfer increase of 20.6 per cent, which is in good agreement with the more detailed analysis in Table 2. The small discrepancy is accounted for in the expansion $(1 - N_{im})^{-1} = 1 + N_{im}$.

The actual heat flux increase may be somewhat larger than shown in Table 2 since two of the reacting species, H and H_2, occur at appreciable mole fractions and have values of $(D/\kappa) \gg 1$. Hence the chemical kinetics may not be sufficiently fast to maintain equilibrium for these species, and it is clear from Equation [6] that, if some of these species react where $D \simeq \epsilon$, the heat flux should be increased somewhat. A detailed investigation of this correction can be made by solving Equations [1] and [2] with suitable expressions for the chemical reaction rates.

IV Conclusions

Heat transfer from flames to cooled surfaces generally can be expected to show an increase because of the heat release resulting from the recombination of dissociated species in the boundary layers and on the surface. For most chemical species the reaction tends to completion in the intermediate layer where D and κ are small with respect to ϵ. Thus, even if $D \neq \kappa$, $(D + \epsilon)/(\kappa + \epsilon) \simeq 1$; and the chemical reaction rates are fast because of the high temperatures encountered

in this region. Under these conditions, a simplified expression may be used for the increase in heat flux which applies regardless of the location of the heat release as long as $D +$

Table 1 Equilibrium flame composition for representative system[a]

Species	Mole fraction	$\Delta H_{formation}$ kcal/mole
H	0.0100	51.6
OH	0.0377	10.0
O	0.0039	58.6
N	0.0002	112.5
H_2	0.0582	0
NO	0.0080	21.5
O_2	0.0162	0

[a] Ammonia (l)–oxygen (g) at 20 atm and stoichiometric mixture ratio.

Table 2 Theoretical increase in heat flux for representative system[a]

Reaction $(A + B$ = products)	Mole fraction of reactants[b] A	B	Heat release Q, kcal/mole	$\Delta q/q'$, %
H + OH =				
$H_2O(g)$	0.0100	0.0100	119.4	4.2
OH + $^1/_2 H_2$ =	0.0277	0.0139	67.8	6.7
$H_2O(g)$	0.0039	0.0039	58.6	0.8
0 + H_2 = H_2O (g)				
NO + H_2 =				
$H_2O(g) + ^1/_2 N_2$	0.0080	0.0080	79.3	2.2
N + N = N_2	0.0002	0.0002	225	0.2
$H_2 + ^1/_2 O_2$ =				
$H_2O(g)$	0.0324	0.0162	57.8	6.6
			Total	20.7

[a] Ammonia (l)–oxygen (g) at 20 atm and stoichiometric mixture ratio; flame temperature 3053 K; wall temperature 600 K; average heat capacity 11.7 cal mole^{-1} °C^{-1} at 2700 K.
[b] Cf. Table 1.

$\epsilon \simeq \kappa + \epsilon$. The theoretical analysis considers also the effect of surface reaction on heat transfer. In this case, certain reactions such as the heterogeneous dissociation of NH_3 may show a decrease in the heat-flux ratio due to the endothermic nature of the reaction.

For the usual case of heat transfer from flames, the concept of the "nondissociated" flame temperature T_2^0 is introduced since it permits a simple, yet reasonably accurate calculation of the change in heat transfer. This concept should be useful in predicting high-temperature, heat-transfer coefficients from low-temperature measurements.

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On Laminar Boundary Layers With Heat Transfer

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In part I, a transformation is given which reduces the laminar boundary layer equations approximately to the incompressible form in the case of an imperfect gas. The transformation is related to the Stewartson-Illingworth transformation, and involves fitting an approximate relation to the density profile. In part II, the problem of utilizing similar solutions in a momentum-integral method is treated with an approach in which the boundary layer thickness is characterized by a single function of distance along the boundary layer; the momentum, displacement, and enthalpy-defect thickness are expressed in dimensionless forms directly calculable from the similar solutions. It is noted that a basic inconsistency appears in the energy relation in that two different expressions for the heat transfer at the wall are obtained if the classic momentum-integral method is used. Various alternate methods are discussed.

Nomenclature

u, v = velocity components
x, y = Cartesian coordinates
$\bar{u}, \bar{v}$ = transformed velocity components
$\bar{x}, \bar{y}$ = transformed coordinates
ρ = density
μ = viscosity
h = enthalpy
H = total enthalpy
σ = Prandtl number
$a(x)$ = transformation function
$N(\bar{y})$ = viscosity ratio function
κ, α = dimensionless parameters
C_p = specific heat at constant pressure
K = coefficient of thermal expansion
T = absolute temperature
$g(x)$ = boundary layer thickness function
β = similarity parameter
f = reduced stream function
η = reduced normal coordinate
Δ = dimensionless momentum thickness
δ = dimensionless displacement thickness
Λ = dimensionless enthalpy-defect thickness
A = ratio of displacement to momentum thickness
Δ^* = logarithmic derivative of Δ with respect to subscript
Λ^* = logarithmic derivative of Λ with respect to subscript
$\bar{\beta}$ = enthalpy similarity parameter
$\Gamma(x)$ = kinetic energy correction parameter
a_1, a_2 = vorticity parameters for exterior flow

Subscripts

1 = free-stream conditions
w = wall conditions

I APPROXIMATE METHOD FOR AN IMPERFECT GAS

Introduction

ANALYTICAL methods for treating laminar boundary layers in a compressible fluid are generally based upon the assumption that the fluid is a perfect gas with constant ratio of specific heats. The first part of this paper presents an approximate method for treating the case of a gas which is not perfect; this method utilizes a velocity transformation similar to the Stewartson-Illingworth transformation (1, 2).[2] The basic Howarth transformation which replaces normal distance by normal mass is included, of course.

The boundary layer equations are taken in their two-dimensional form; the modification required for the axisymmetric case is indicated later.

Basic Equations

The basic boundary-layer equations for a two-dimensional laminar boundary layer may be expressed

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\rho_1}{\rho} u_1 \frac{du_1}{dx} + \frac{1}{\rho} \frac{\partial}{\partial y}\left[\mu \frac{\partial u}{\partial y}\right] \dots \text{[I-1a]}$$

$$u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y}\left[\frac{\mu}{\sigma}\left(\frac{\partial H}{\partial y} + \frac{\sigma-1}{2} \frac{\partial u^2}{\partial y}\right)\right] \dots \text{[I-1b]}$$

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \dots \text{[I-1c]}$$

In these equations the total enthalpy H is given by

$$H = h + \tfrac{1}{2} u^2 \dots \text{[I-2]}$$

and the viscous shear and conductive heat transfer are $\mu(\partial u/\partial y)$ and $(\mu/\sigma)(\partial h/\partial y)$, respectively.

The quantity N is introduced, defined by

$$N = \frac{\rho\mu}{\rho_w \mu_w} \dots \text{[I-3]}$$

Using this terminology, the quantity $\rho_w\mu_w$ may be taken outside the derivative in the last terms of Equations [I-1a] and [I-1b]. A variant of the Stewartson-Illingworth transformation is now introduced, through a function $a(x)$ which is as yet undetermined. A reduced longitudinal velocity component $\bar{u}$ is introduced

$$u = a\bar{u} \dots \text{[I-4]}$$

in terms of which the basic equations are rewritten

$$\bar{u} \frac{\partial \bar{u}}{\partial x} + \frac{v}{a} \frac{\partial \bar{u}}{\partial y} = \frac{\rho_1}{\rho} \frac{u_1}{a^2} \frac{du_1}{dx} - \frac{\bar{u}^2}{a} \frac{da}{dx} + \frac{\rho_w\mu_w}{a\rho} \frac{\partial}{\partial y}\left[\frac{N}{\rho} \frac{\partial \bar{u}}{\partial y}\right] \dots \text{[I-5a]}$$

$$\bar{u} \frac{\partial H}{\partial x} + \frac{v}{a} \frac{\partial H}{\partial y} = \frac{\rho_w\mu_w}{a\rho} \frac{\partial}{\partial y}\left[\frac{N}{\sigma\rho}\left(\frac{\partial H}{\partial y} + \frac{(\sigma-1)a^2}{2} \frac{\partial \bar{u}^2}{\partial y}\right)\right] \dots \text{[I-5b]}$$

$$\frac{\partial a\rho\bar{u}}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \dots \text{[I-5c]}$$

The quantity κ is now defined, relating the functions $u_1(x)$ and $a(x)$

$$\kappa = \frac{u_1}{\bar{u}_1}\frac{d\bar{u}_1}{du_1} = 1 - \frac{u_1}{a}\frac{da}{du_1}\ldots\ldots\ldots\ldots [I\text{--}6]$$

The quantity κ is a function of x, in general. A dimensionless variable θ is introduced to replace H, through a parameter α which will be assumed to be constant

$$\frac{H}{H_1} = \alpha + (1 - \alpha)\theta\ldots\ldots\ldots\ldots [I\text{--}7]$$

It may be noted that $\theta_1 = 1$.

An approximate relation connecting the density ratio and the variable θ is now postulated

$$\frac{\rho_1}{\rho} + (\kappa - 1)\frac{u^2}{u_1^2} = \kappa\theta\ldots\ldots\ldots\ldots [I\text{--}8]$$

From Equations [I–6], [I–7], and [I–8] may be obtained the relation

$$\frac{\rho_1}{\rho}\frac{u_1}{a^2}\frac{du_1}{dx} - \frac{\bar{u}^2}{a}\frac{da}{dx} = \theta\bar{u}_1\frac{d\bar{u}_1}{dx}\ldots\ldots\ldots\ldots [I\text{--}9]$$

The transformation is now completed with the introduction of the transformed quantities

$$\bar{x} = \int a\rho_w\mu_w\,dx\ldots\ldots\ldots\ldots [I\text{--}10a]$$

$$\bar{y} = a\int \rho\,dy\ldots\ldots\ldots\ldots [I\text{--}10b]$$

$$\bar{v} = \frac{1}{a\rho_w\mu_w}\left(\rho v + \bar{u}\int \frac{\partial a\rho}{\partial x}dy\right)\ldots\ldots\ldots [I\text{--}10c]$$

The basic equations now take the form

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \theta\bar{u}_1\frac{d\bar{u}_1}{d\bar{x}} + \frac{\partial}{\partial \bar{y}}\left[N\frac{\partial \bar{u}}{\partial \bar{y}}\right]\ldots\ldots [I\text{--}11a]$$

$$\bar{u}\frac{\partial \theta}{\partial \bar{x}} + \bar{v}\frac{\partial \theta}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}}\left[\frac{N}{\sigma}\left(\frac{\partial \theta}{\partial \bar{y}} + \frac{(\sigma - 1)a^2}{2(1 - \alpha)H_1}\frac{\partial \bar{u}^2}{\partial \bar{y}}\right)\right]\ldots [I\text{--}11b]$$

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0\ldots\ldots\ldots\ldots [I\text{--}11c]$$

Except for the term in $(\sigma - 1)$ in Equation [I–11b], these equations are now in the low-velocity form for a perfect gas.

Fitting the Density Ratio

The application of the foregoing transformation requires the establishment of the approximate validity of Equation [I–8]. This relation is automatically satisfied under free-stream conditions. It is now required that it be satisfied at the wall and in its first derivative at the wall.

Using Equation [I–7], the condition at the wall is expressed

$$\frac{h_w}{H_1} = \alpha + \frac{(1 - \alpha)}{\kappa}\frac{\rho_1}{\rho_w}\ldots\ldots\ldots\ldots [I\text{--}12]$$

The coefficient of thermal expression K is introduced

$$K = \rho\left[\frac{\partial(1/\rho)}{\partial T}\right]_p\ldots\ldots\ldots\ldots [I\text{--}13]$$

in terms of which the condition on the derivative is expressed

$$\frac{C_{pw}}{H_1} = \frac{(1 - \alpha)}{\kappa}\frac{\rho_1}{\rho_w}K_w\ldots\ldots\ldots\ldots [I\text{--}14]$$

Eliminating κ yields

$$\alpha \quad \frac{h_w - \dfrac{C_{pw}}{K_w}}{H_1}\ldots\ldots\ldots\ldots [I\text{--}15]$$

whence κ may be expressed

$$\kappa = \frac{\rho_1}{\rho_w}\left[1 + \frac{H_1 - h_w}{C_{pw}/K_w}\right]\ldots\ldots\ldots\ldots [I\text{--}16]$$

It is presumed that α as given by Equation [I–15] is approximately constant. If this is not so, a suitable value must be chosen and the fitting accomplished by obtaining κ from Equation [I–12].

An important special case is that for which the gas is imperfect in general, but under the conditions at and near the wall behaves like a perfect gas with constant ratio of specific heats. For such a perfect gas $K = T^{-1}$, and $C_p/K = h$. In this case the parameter α as derived from Equation [I–15] is zero, and

$$\kappa = \frac{\rho_1}{\rho_w}\frac{H_1}{h_w}; \qquad \alpha = 0\ldots\ldots\ldots\ldots [I\text{--}17]$$

The case for which the gas is perfect throughout serves as a check on these results. In this case $\kappa = H_1/h_1$, and Equation [I–6] may be integrated to give a proportional to $h_1^{1/2}$. The transformation in this case is identical with that of Stewartson (1) and Illingworth (2), for which Equation [I–8] is exact.

Axisymmetric Case

For a body of revolution at zero incidence for which the body curvatures times boundary-layer thickness are small, the well-known Mangler transformation may be used to reduce the equations to the two-dimensional form.

A shape function $r(x)$ appears as a factor within the differentials of the continuity equation Equations [I–1c], [I–5c]. The Mangler transformation may be included in the transformation of Equations [I–10] by replacing a by ar, $\rho_w\mu_w$ by $\rho_w\mu_w r$, and v by vr in those equations. The set of Equations [I–11] then follow from the altered set Equations [I–5].

Forward Stagnation Point

The density ratio only plays a part in determining the boundary layer solution if the pressure gradient is different from zero. To exemplify the method presented here, the case of a forward stagnation point is considered. The function g and the parameter β are defined

$$g^2 = 2\int \bar{u}_1\,d\bar{x}_1\ldots\ldots\ldots\ldots [I\text{--}18a]$$

$$\beta = \frac{g^2}{\bar{u}_1^2}\frac{d\bar{u}_1}{d\bar{x}}\ldots\ldots\ldots\ldots [I\text{--}18b]$$

and a stream function gf and a reduced normal coordinate η are introduced such that

$$\frac{\partial gf}{\partial \bar{y}} = \bar{u}\ldots\ldots\ldots\ldots [I\text{--}19a]$$

$$\eta = \frac{\bar{u}_1}{g}\bar{y}\ldots\ldots\ldots\ldots [I\text{--}19b]$$

The $(\sigma - 1)$ term of Equation [I–11b] is dropped, β and θ_w are assumed to be constant, and N and σ are assumed to be functions of η alone. Under these conditions, similar boundary layer profiles may be obtained, corresponding to the solution of the ordinary differential equations

$$(Nf_{\eta\eta})_\eta + ff_{\eta\eta} + \beta(\theta - f_\eta^2) = 0\ldots\ldots\ldots [I\text{--}20a]$$

$$\left(\frac{N}{\sigma}\theta_\eta\right)_\eta + f\theta_\eta = 0\ldots\ldots\ldots\ldots [I\text{--}20b]$$

These are essentially the similar solutions found by Stewartson (1) and treated in detail with extensive calculations for the

case ($\sigma = 1$, $N = 1$) by Cohen and Reshotko (3). These solutions have also been studied by Li and Nagamatsu (4) and by Levy (5). Cohen's quantity S is equal to $\theta - 1$ here.

In the vicinity of a forward stagnation point the quantity κ may be assumed constant and the relations

$$\bar{u}_1 \propto u_1^\kappa \dots\dots\dots\dots\dots \text{[I–21a]}$$

$$\bar{x} \propto x^{1-\kappa} \dots\dots\dots\dots\dots \text{[I–21b]}$$

established locally. The resulting similar profile is one for which

$$\beta = \kappa \dots\dots\dots\dots\dots \text{[I–22]}$$

instead of $\beta = 1$, as in the perfect-gas case.

In the axisymmetric case the influence of the Mangler transformation with $r \doteq x$ is to change Equation [I–21b] to

$$\bar{x} \propto x^{1-\kappa} \dots\dots\dots\dots\dots \text{[I–23]}$$

leading to the value

$$\beta = \kappa/2 \dots\dots\dots\dots\dots \text{[I–24]}$$

in place of $\beta = {}^1/_2$, as in the perfect-gas case.

In either the two-dimensional or the axisymmetric case, then, the fact that the gas is imperfect may be approximately taken into account for a forward stagnation point through a change in the similarity parameter β of the boundary-layer profile.

Shear Stress and Heat Transfer

In order to reconnect the physical quantities of interest with the analysis presented, the shear stress $\mu(\partial u/\partial y)$ and the heat transfer $(\mu/\sigma)(\partial h/\partial y)$ are expressed in terms of the quantities introduced. The shear stress is given

$$\mu \frac{\partial u}{\partial y} = \rho \mu a^2 \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\rho \mu u_1^2}{g} f_{\eta\eta} \dots\dots\dots \text{[I–25a]}$$

$$\left(\mu \frac{\partial u}{\partial y} \right)_w = \rho_w \mu_w a^2 \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)_w = \frac{\rho_w \mu_w u_1^2}{g} f_{\eta\eta_w} \dots \text{[I–25b]}$$

The heat transfer is given

$$\frac{\mu}{\sigma} \frac{\partial h}{\partial y} + \frac{u}{\sigma} \left(\mu \frac{\partial u}{\partial y} \right) = \frac{\rho \mu H_1 (1 - \alpha) a}{\sigma} \frac{\partial \theta}{\partial \bar{y}} = \frac{\rho \mu H_1 (1 - \alpha) u_1}{\sigma g} \theta_\eta$$
$$\dots\dots \text{[I–26a]}$$

$$\left(\frac{\mu}{\sigma} \frac{\partial h}{\partial y} \right)_w = \frac{\rho_w \mu_w H_1 (1 - \alpha) a}{\sigma_w} \left(\frac{\partial \theta}{\partial \bar{y}} \right)_w = \frac{\rho_w \mu_w H_1 (1 - \alpha) u_1}{\sigma_w g} \theta_{\eta_w}$$
$$\dots\dots \text{[I–26b]}$$

In the axisymmetric case the factor r appears in these equations.

II MOMENTUM-INTEGRAL METHODS

Introduction

The calculation of laminar boundary layers with an arbitrary pressure distribution is generally carried out with one of various assumptions as to the profiles which may be present, utilizing equations involving integrals taken across the boundary layer. The primary equation considered is that derived from the momentum equation which involves the momentum and displacement thicknesses. It is convenient to assume that the boundary layer profiles at any point are the same as those for one of a family of similar solutions for which separate calculations are available. This second part of the paper presents a modification of the classical momentum-integral method in which the various integral thicknesses are used in the dimensionless forms which arise naturally from the similar solutions.

Application of similar solutions to calculations with arbitrary pressure distributions was first made by Falkner and Skan (6) in a method which did not use the momentum-inte-

gral concept. The two concepts were combined by Thwaites (7) who assumed a correlation connecting the shear stress at the wall, the local pressure gradient, and the ratio of displacement to momentum thickness, and whose selection of a particular correlation was influenced by available calculations of similar solutions. The approach of Thwaites has been extended by Rott and Crabtree (8) who treated in particular the case $\sigma = 1$, $\theta_w = 1$, and by Cohen and Reshotko (9) who treated in particular the case $\sigma = 1$, $\theta_w = $ const. For detailed discussion and extensive references, the reader is referred to the authors cited.

The approach of this note is similar in many respects to the Thwaite approach; it differs in that the basic boundary layer thickness used as a reference value is a function $g(x)$ appearing in the separation-of-variables procedure underlying the similar solution analysis rather than the related quantity of Thwaites defined in terms of the momentum thickness.

Basic Equations

The laminar boundary layer equations will be considered in their fully reduced form, as given in Equations [I–11]. The bars have been dropped here, so that the fact that u, v, x, and y are reduced quantities must be kept in mind.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \theta u_1 \frac{du_1}{dx} + \frac{\partial}{\partial y} \left[N \frac{\partial u}{\partial y} \right] \dots \text{[II–1a]}$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{\partial}{\partial y} \left[\frac{N}{\sigma} \left(\frac{\partial \theta}{\partial y} + \frac{\sigma - 1}{2 u_1^2} \Gamma \frac{\partial u^2}{\partial y} \right) \right] \dots \text{[II–1b]}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \dots\dots\dots\dots \text{[II–1c]}$$

where

$$\Gamma(x) = \frac{u_1^2 a^2}{(1 - \alpha) H_1} \dots\dots\dots\dots \text{[II–2]}$$

is a parameter equal to twice the ratio of kinetic energy to total enthalpy in the free stream. The boundary conditions are

$$y = 0: \quad u = v = 0, \quad \theta = \theta_w(x), \quad (N = 1) \dots \text{[II–3a]}$$

$$y = \infty: \quad u = u_1(x), \quad \theta = 1 \dots\dots\dots \text{[II–3b]}$$

A function $g(x)$ which is as yet undetermined is introduced, together with a stream function for Equation [II–1c] equal to gf. The variable y is replaced by a variable η defined by

$$\eta = \frac{u_1}{g} y \dots\dots\dots\dots\dots \text{[II–4]}$$

With these quantities Equations [II–1] become

$$(N f_{\eta\eta})_\eta + \frac{g'g}{u_1} f f_{\eta\eta} + \frac{g^2 u_1'}{u_1^2} (\theta - f_\eta^2) = \frac{g^2}{u_1} (f_\eta f_{\eta x} - f_{\eta\eta} f_x)$$
$$\dots\dots \text{[II–5a]}$$

$$\left(\frac{N}{\sigma} \theta_\eta \right)_\eta + \frac{g'g}{u_1} f \theta_\eta = \frac{g^2}{u_1} (f_\eta \theta_x - \theta_\eta f_x) - \left(\frac{\sigma - 1}{\sigma} N \Gamma f_\eta f_{\eta\eta} \right)_\eta$$
$$\dots\dots \text{[II–5b]}$$

under the boundary conditions

$$\eta = 0: \quad f = 0, \quad f_\eta = 0, \quad \theta = \theta_w(x), \quad (N = 1)$$
$$\dots\dots \text{[II–6a]}$$

$$\eta = \infty: \quad f_\eta = 1, \theta = 1 \dots\dots\dots\dots \text{[II–6b]}$$

An integral of Equations [II–5] from $\eta = 0$ to ∞ is now taken, giving

$$-f_{\eta\eta_w} + \left(\frac{g'g}{u_1} + \frac{g^2 u_1'}{u_1^2} \right) \Delta + \frac{g^2 u_1'}{u_1^2} \delta = -\frac{g^2}{u_1} \frac{d\Delta}{dx} \dots \text{[II–7a]}$$

$$-\frac{1}{\sigma_w} \theta_{\eta_w} + \frac{g'g}{u_1} \Lambda = -\frac{g^2}{u_1} \frac{d\Lambda}{dx} \dots\dots\dots \text{[II–7b]}$$

where

$$\Delta = \int_0^\infty f_\eta (1 - f_\eta)\, d\eta \quad \ldots\ldots\ldots [\text{II--8a}]$$

$$\delta = \int_0^\infty (\theta - f_\eta)\, d\eta = A\Delta \quad \ldots\ldots [\text{II--8b}]$$

$$\Lambda = \int_0^\infty f_\eta (1 - \theta)\, d\eta \quad \ldots\ldots\ldots [\text{II--8c}]$$

are dimensionless momentum, displacement, and enthalpy-defect thicknesses, respectively. Equations [II–7] may be rewritten

$$\frac{g^2}{u_1}\frac{d\Delta}{dx} + \left[\frac{g'g}{u_1} + (1 + A)\frac{g^2 u_1'}{u_1{}^2}\right]\Delta = f_{\eta\eta_w} \ldots [\text{II--9a}]$$

$$\frac{g^2}{u_1}\frac{d\Lambda}{dx} + \left(\frac{g'g}{u_1}\right)\Lambda = \frac{1}{\sigma_w}\theta_{\eta_w} \ldots\ldots\ldots [\text{II--9b}]$$

Similar Solutions

A solution to the set of equations

$$(Nf_{\eta\eta})_\eta + ff_{\eta\eta} + \beta(\theta - f_\eta{}^2) = 0 \ldots\ldots [\text{II--10a}]$$

$$\left(\frac{N}{\sigma}\theta_\eta\right)_\eta + fc_\eta + \left(\frac{\sigma-1}{\sigma}N\Gamma f_\eta f_{\eta\eta}\right)_\eta = 0 \ldots [\text{II--10b}]$$

considered as ordinary equations in η is here defined as a similar solution. Such a solution corresponds to a physically realizable similar solution which is an x-independent solution to the boundary layer equations only if N and σ are functions of η alone and Γ is constant or $\sigma = 1$. It is now assumed that the variables Δ, A, and $f_{\eta\eta_w}$ in Equation [II–9a] and Λ and θ_{η_w} in Equation [II–9b] are the same as they would be if the actual solution were a similar one with the local values of θ_w and Γ and with a suitable value of β. This is the underlying assumption of this method and corresponds to Thwaites' correlation assumption. With this assumption, the relations

$$[1 + (1 + A)\beta]\Delta = f_{\eta\eta_w} \ldots\ldots\ldots\ldots [\text{II--11a}]$$

$$\Lambda = \frac{1}{\sigma_w}\theta_{\eta_w} \ldots\ldots\ldots\ldots [\text{II--11b}]$$

are valid, and Equations [II–9] may be put in the form

$$\frac{g^2}{u_1\Delta}\frac{d\Delta}{dx} + \frac{g'g}{u_1} - 1 + (1 + A(\beta))\left(\frac{g^2 u_1'}{u_1{}^2} - \beta\right) = 0 \ldots [\text{II--12a}]$$

$$\frac{g^2}{u_1\Lambda}\frac{d\Lambda}{dx} + \frac{g'g}{u_1} - 1 = 0 \ldots\ldots\ldots [\text{II--12b}]$$

The quantities Δ and Λ are now considered to be functions of β and θ_w alone, for the purpose of simplifying the analysis. Dependence upon Γ or σ_w may be treated in the same way as dependence upon θ_w, while variations in $N(\eta)$ or in σ/σ_w with x would entail some additional complication. Logarithmic partial derivatives are defined

$$\Delta^*{}_\beta = \frac{\beta}{\Delta}\frac{\partial\Delta}{\partial\beta}; \quad \Delta^*{}_{\theta_w} = \frac{\theta_w}{\Delta}\frac{\partial\Delta}{\partial\theta_w} \ldots\ldots [\text{II--13a}]$$

$$\Lambda^*{}_\beta = \frac{\beta}{\Lambda}\frac{\partial\Lambda}{\partial\beta}; \quad \Lambda^*{}_{\theta_w} = \frac{\theta_w}{\Lambda}\frac{\partial\Lambda}{\partial\theta_w} \ldots\ldots [\text{II--13b}]$$

with respect to β and θ_w; similar notation may be used with respect to Γ and σ_w. Equations [II–12] now becomes

$$\frac{g^2}{u_1}\left(\frac{\beta'}{\beta}\Delta^*{}_\beta + \frac{\theta_w'}{\theta_w}\Delta^*{}_{\theta_w}\right) +$$
$$\frac{g'g}{u_1} - 1 + (1 + A)\left(\frac{g^2 u_1'}{u_1{}^2} - \beta\right) = 0 \ldots [\text{II--14a}]$$

$$\frac{g^2}{u_1}\left(\frac{\beta'}{\beta}\Lambda^*{}_\beta + \frac{\theta_w'}{\theta_w}\Lambda^*{}_{\theta_w}\right) + \frac{g'g}{u_1} - 1 = 0 \ldots [\text{II--14b}]$$

The functions $u_1(x)$ and $\theta_w(x)$ are known functions, while $g(x)$ and $\beta(x)$ are as yet unknown. Some latitude is available in the manner in which g and β are determined.

The Classical Method

In the classical method of applying the momentum-integral method, the enthalpy equation is ignored and an assumption is made regarding the relation between the basic parameter and the velocity gradient. As interpreted here, Equation [II–14b] is ignored and β is determined by the relation

$$\beta = \frac{g^2 u_1'}{u_1{}^2} \ldots\ldots\ldots\ldots [\text{II--15}]$$

Equation [II–14a] gives the differential equation for g

$$\frac{g'g}{u_1}(1 + 2\Delta^*{}_\beta) - \beta\left(2 - \frac{u_1 u_1''}{u_1'{}^2}\right)\Delta^*{}_\beta - 1 + \frac{g^2}{u_1}\frac{\theta_w'}{\theta_w}\Delta^*{}_{\theta_w} = 0$$
$$\ldots\ldots [\text{II--16}]$$

or the equivalent differential equation for β

$$\frac{\beta'}{\beta}(1 + 2\Delta^*{}_\beta) - 2\frac{(1 - \beta)}{\beta}\frac{u_1'}{u_1} - \frac{u_1''}{u_1'} + 2\frac{\theta_w'}{\theta_w}\Delta^*{}_{\theta_w} = 0$$
$$\ldots\ldots [\text{II--17}]$$

Either of these equations may be integrated numerically to obtain the quantities g and β.

The heat transfer may be calculated from the momentum solution in two different ways. The relation Equation [II–11b] may be used, corresponding to the slope of the assumed local enthalpy profile corresponding to the velocity profile; or, the relation Equation [II–9b] may be used, corresponding to the heat transfer necessary to maintain the assumed corresponding enthalpy profile. These give the heat transfer equations

$$\theta_{\eta_w} = \sigma_w\Lambda \ldots\ldots\ldots\ldots [\text{II--18a}]$$

$$\theta_{\eta_w} = \sigma_w\Lambda\left[1 - \frac{g^2}{u_1}\frac{\beta'}{\beta}(\Delta^*{}_\beta - \Lambda^*{}_\beta) - \frac{g^2}{u_1}\frac{\theta_w'}{\theta_w}(\Delta^*{}_{\theta_w} - \Lambda^*{}_{\theta_w})\right]$$
$$\ldots\ldots [\text{II--18b}]$$

These equations cannot be consistent in general unless Δ equals a constant times Λ over the entire range of the parameters. It is not completely clear which of Equations [II–18] should be preferable, although it appears to the writer that the simpler form Equation [II–18a] should be more trustworthy.

A basic inconsistency in the calculation of heat transfer from the enthalpy profile corresponding to the assumed velocity profile thus appears. That this is so is not surprising in light of the fact that the upstream temperature profile has no effect on the heat transfer calculated by this method, except in so far as it may affect the velocity profile.

Alternate Methods

A. In this method the relation Equation [II–15] connecting β and g is dropped, and Equations [II–14] are to be solved as a pair of simultaneous equations for determining both β and g. The derivatives of β and g may be separated, giving the equations

$$\frac{g^2\beta'}{u_1\beta}(\Delta^*{}_\beta - \Lambda^*{}_\beta) +$$
$$\frac{g^2\theta_w'}{u_1\theta_w}(\Delta^*{}_{\theta_w} - \Lambda^*{}_{\theta_w}) + (1 + A)\left(\frac{g^2 u_1'}{u_1{}^2} - \beta\right) = 0 \ldots [\text{II--19a}]$$

$$\left(\frac{g'g}{u_1} - 1\right)(\Delta^*{}_\beta - \Lambda^*{}_\beta) +$$
$$\frac{g^2\theta_w'}{u_1\theta_w}(\Delta^*{}_\beta\Lambda^*{}_{\theta_w} - \Lambda^*{}_\beta\Delta^*{}_{\theta_w}) - (1 + A)\left(\frac{g^2 u_1'}{u_1{}^2} - \beta\right)\Delta^*{}_\beta = 0$$
$$\ldots [\text{II--19b}]$$

This set of equations may pose difficulties of a mathematical nature, particularly with regard to the question of stability. An approximate statement of a stability condition may be formulated, that

$$\Delta^*{}_\beta - \Lambda^*{}_\beta < 0 \ldots \ldots \ldots \ldots [\text{II-20}]$$

This method gives a simultaneous solution for the skin friction and the wall heat transfer, in a self-consistent form, provided that a mathematically unobjectionable solution can be found.

B. The classical method as just described may be followed except for the computation of heat transfer properties. The enthalpy profile is now assumed to be a similar one with the appropriate values of θ_w and Γ with the function $g(x)$ computed for the momentum equation, but with a different value of the fundamental parameter $\bar{\beta}$, corresponding to a different value of $\bar{\Lambda}$. This gives the equation for $\bar{\beta}$ in place of Equation [II-14b]

$$\frac{\bar{\beta}'}{\bar{\beta}}\bar{\Lambda}^*{}_{\bar{\beta}} + \frac{\theta_w'}{\theta_w}\bar{\Lambda}^*{}_{\theta_w} = \frac{u_1}{g^2}\left(1 - \frac{g'g}{u_1}\right)$$
$$= \frac{\beta'}{\beta}\Delta^*{}_\beta + \frac{\theta_w'}{\theta_w}\Delta^*{}_{\theta_w} \ldots \ldots [\text{II-21}]$$

The heat transfer is given directly in terms of $\bar{\Lambda}$. This method may be expected to give reasonably trustworthy results if the difference between β and $\bar{\beta}$ does not become too great.

C. For a very rough estimate all the derivatives of Δ and Λ in Equations [II-14] may be neglected, giving the relation Equation [II-15] for β in terms of g, and the relation

$$g^2 = 2 \int u_1 \, dx \ldots \ldots \ldots \ldots [\text{II-22}]$$

Although rather crude, this method should give an estimate of the heat transfer from Equation [II-11b] which is about as accurate as that given by the classical method.

D. The solution obtained by method C may be used to calculate the Δ and Λ derivative terms in Equations [II-14] for use in an algebraic calculation of second approximation. Such a successive approximation scheme, although not strictly convergent, may have semiconvergent properties that make it useful if $|\Delta^*{}_\beta - \Lambda^*{}_\beta|$ is small. This should give essentially the solution of method A by an easier method.

External Rotational Flow

One of the basic boundary layer assumptions is that the layer is so thin that the structure of the external flow does not affect the structure of the boundary layer except through the single function $u_1(x)$. It is possible that in certain cases, if the vorticity in the external flow is sufficiently large, that this vorticity may influence the boundary layer structure even though the layer is quite thin. For a similar boundary layer profile similarity conditions must be imposed on the external vorticity which are, in general, inconsistent with the steady-flow conditions on the inviscid exterior flow. However, the case of the neighborhood of the stagnation point on a blunt body of revolution is one in which finite vorticity at the wall does follow the similarity condition. With the type of use of similar profiles of this note the requirement of the similarity condition on the vorticity is not essential.

The inviscid profile (no boundary layer present) is assumed to be of the form

$$f_\eta = 1 + a_1\eta + a_2\eta^2 + \ldots \ldots \quad \ldots \ldots [\text{II-26}]$$

The procedure to be followed is first to form the inviscid expression for f

$$f = \eta + \tfrac{1}{2}a_1\eta^2 + \tfrac{1}{3}a_2\eta^3 + \ldots \ldots \quad \ldots \ldots [\text{II-27}]$$

and then eliminate η between Equations [II-26] and [II-27]. The resultant relation connecting f_η and f then replaces the boundary condition $f_\eta = 1$ of Equation [II-6b]. With only the quantity a_1 present or important, this relation is

$$f_\eta{}^2 = 1 + 2a_1 f \ldots \ldots \ldots \ldots [\text{II-28}]$$

The absolute magnitude of the parameter a_1 serves as a measure of its importance in affecting the boundary layer profile. Unless a_1 is negligibly small, it should be added as a parameter determining the boundary layer solution.

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Laminar Heat Transfer Over Blunt-Nosed Bodies

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Technical Notes

Sun-Follower for High Altitude Sounding Rocket

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A PROGRAM of upper atmosphere research has required the direct measurement of solar radiation below 3100 Ångstroms and atmospheric absorption of this radiation. Units have been designed, built, and flown which house a spectrograph and direct the solar radiation into this spectrograph. These units, known as Sun-Followers, are mounted on high altitude sounding rockets. In order to obtain sufficient exposure times at the shorter wave lengths, sunlight is directed into the spectrograph regardless of the spin and precession of the rocket. In the latest equipment this is accomplished by means of a two-axis tracker, design and performance of which are described in this report.

The present design of the Sun-Follower has been developed as a result of the experience gained on several previous units built and tested. The first units were single axis units, tracking the sun in azimuth only, and were mounted on a V-2 and on an Aerobee. A subsequent pair of units was designed to mount on a Viking rocket and an Aerobee. These units were also single axis types but incorporated design improvements.

In order to obtain more intense spectra the latest units track the sun in both axes. This is accomplished by two-axis control of a mirror which reflects the sunlight down the longitudinal axis of the rocket, on which the spectrograph is fixed. An azimuth drive corrects for rocket roll motion and an elevation drive corrects for rocket pitch and yaw motion. Photoelectric cells mounted on the line of sight are used to sense angular pointing errors and, by means of servo systems, the error is kept to a minimum.

Figs. 1, 2, and 3 show some of the construction details of the latest Sun-Followers. A block diagram of the control system is shown in Fig. 4.

Photoelectric Cells

For control of elevation and azimuth axes, two pairs of photoelectric cells are employed; one pair being used for coarse control to search for the sun, lock on, and to bring the Sun-Follower approximately in line with the sun, the second pair of photoelectric cells to keep the Sun-Follower accurately on the sun to within ±0.25 deg. A special switching circuit is used to remove the supply voltage to the coarse photoelectric cells when the Sun-Follower is operating on the fine eye system.

The coarse eyes for azimuth control are mounted on the azimuth ring as shown in Fig. 3. The fine azimuth and fine and coarse elevation eyes are mounted in an eye block which is driven by the elevation servo to the line of sight (Fig. 3). The mirror is geared directly to this eye block but is driven at one-half speed in order to obtain the reflection on the longitudinal axis.

Each photoelectric cell is followed by a cathode follower, and the pointing errors are obtained as the differential out-
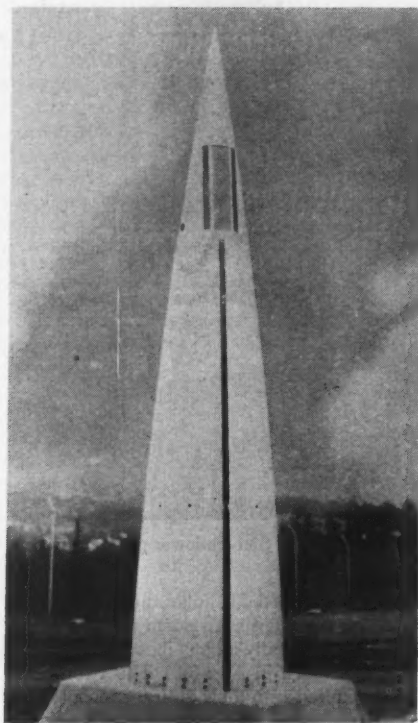
Fig. 1 Assembly of Sun-Follower including rocket skin sections
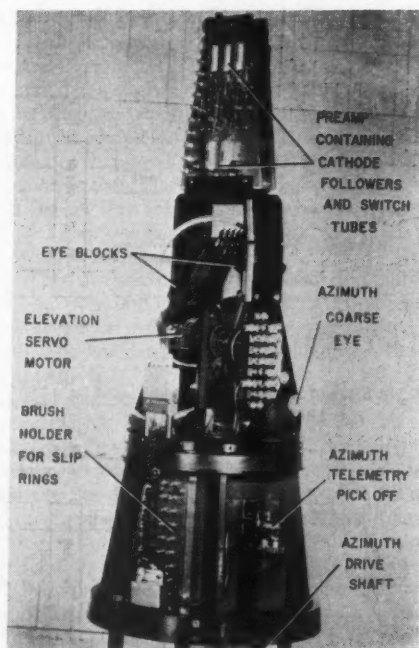
Fig. 2 Internal details of Sun-Follower

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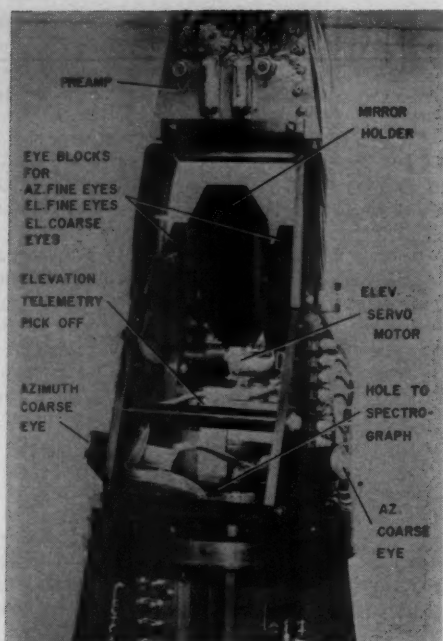
Fig. 3   Details of Sun-Follower photoelectric eyes
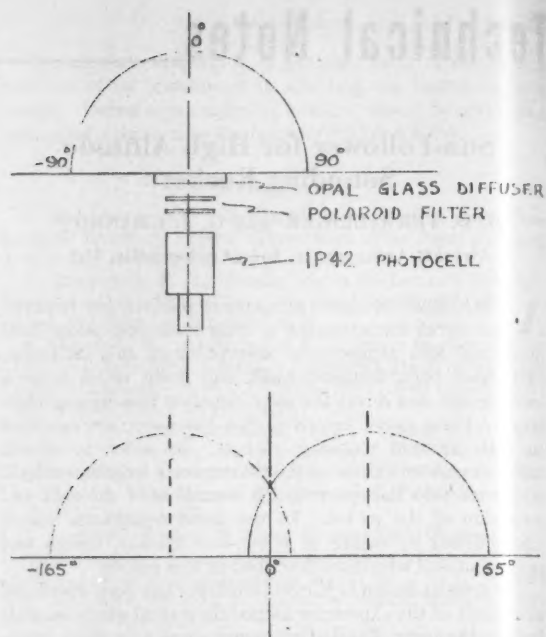
Fig. 5a, b   Coarse eye assembly and pickup patterns

puts of the pairs of cathode followers. The coarse photoelectric cells use an optical system to produce a pickup pattern which is essentially a half sinusoid with the maximum dead ahead, as shown in Fig. 5a. The two coarse eyes in azimuth are mounted with their optical axes located ±75 deg from the boresight line. In this manner a composite error curve as shown in Fig. 5b is obtained.

This permits a proper servo-error signal to be obtained from +165 deg to −165 deg. The remaining 30 deg directly opposite from the sun is not covered, but unbalance of the servo system, stray light, or slow roll of the rocket should

bring the system out of this possible dead zone. The elevation coarse eyes are boresighted along the line-of-sight. A blinder for each eye is provided to distort the pickup pattern to give a large signal on either side of dead ahead. This results in a relatively broad null but the fine eyes are capable of controlling the Sun-Follower in this region. As elevation coverage is only required to be ±45 deg, no dead zone exists.

The fine photoelectric cells use an optical system to obtain a pickup pattern as shown in Fig. 6. The design of the fine and coarse eye assemblies and the eyes used on this model Sun-Follower were supplied by the University of Colorado.
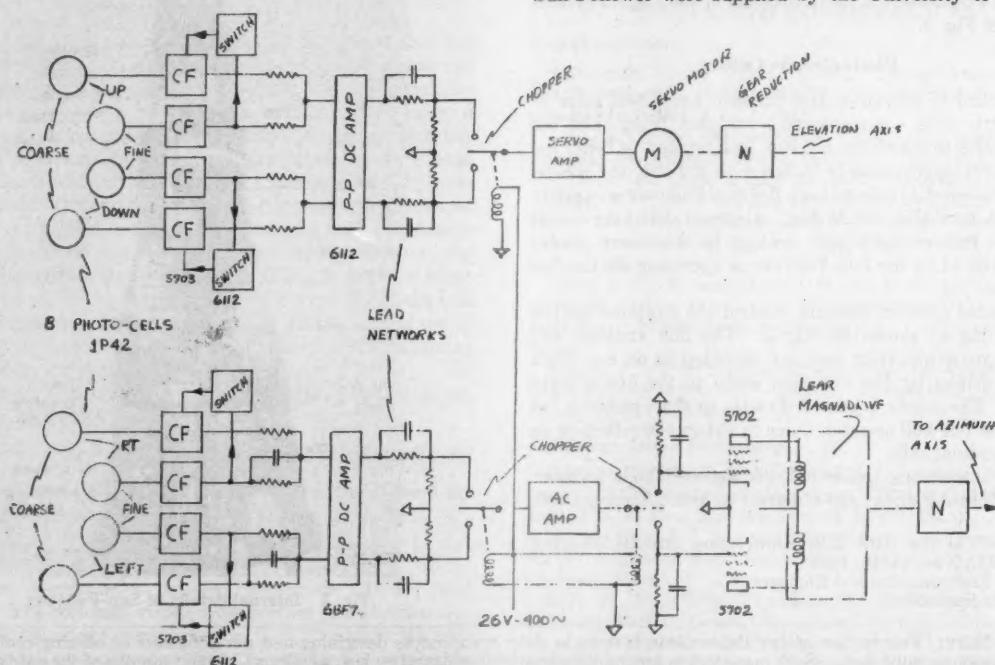
Fig. 4   Block diagram of control circuitry

These eyes were developed by them for use on their Biaxial Pointing Control System (Special Report No. 7, "Photoelectric Eyes for a Solar Pointing Control"). These are arranged so that one photoelectric cell has a pattern as shown by the solid line, to the right of center, and the other photo-

LENS
KNIFE EDGE AT FOCUS OF LENS
POLAROID FILTER
IP42 PHOTOCELL
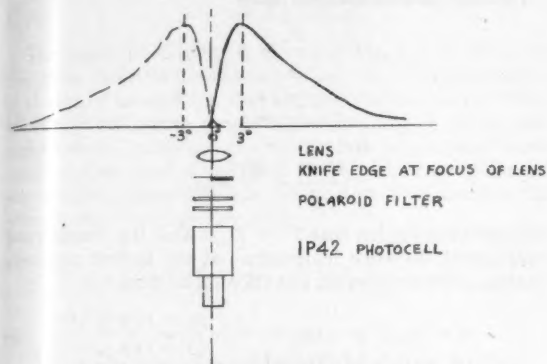
Fig. 6 Fine eye assembly and pickup pattern

electric cell of the pair has the mirror image on the left of center as shown by the dashed line. Similarly, for elevation, the fine eye pair is arranged above and below center. The combination of lens and knife edge at the focal point of the lens gives the sharp cutoff on the unwanted side of center, and the steep linear rise on the other.

Coarse to Fine Switching Circuitry

A special switching circuit was developed to perform two important functions. The first was to switch from coarse to fine eye control. The second was to disable the coarse eyes once the fine eyes were functioning. This is required in order to prevent coercion of the fine data during tracking.

The possibility of coercion exists because the coarse azimuth eyes are mounted on the azimuth axis while the fine eyes are mounted on the line of sight or deflection axis. Any misalignment of the coarse eyes as a function of elevation will cause a coercive error. Also extraneous sources of light such as reflections from the earth or clouds may cause errors in the coarse system. In one of the previous rocket firings, the telemetered data indicated that this actually happened.

In the present switching circuitry, the fine eye photoelectric cell cathode follower outputs go to the grids of a dual triode, the plates of which are tied together. If either of the fine eye photoelectric cells are illuminated, the grids of the switch tube are driven sufficiently positive to saturate the tube and to bring its plate potential to a low value. The coarse eye photoelectric cell anodes are connected to the switch tube plates, so that when the fine eyes are illuminated, the coarse eye photoelectric cell anode potential drops to a sufficiently low value that the coarse eyes are inoperative regardless of the illumination on these cells.

The outputs of the coarse and fine systems are mixed as shown in Fig. 4, by means of adding resistors, with only one system operative at any one time.

Servo Amplifier

Following the mixing network, the differential signals are amplified, passed through a lead network for stabilization of the servo loop, and modulated by means of a 400 cps chopper. In the elevation system, this a-c error signal is amplified to drive the elevation servo motor which in turn drives the elevation axis. In the azimuth system, the a-c error signal is amplified, demodulated by a synchronous chopper, and fed to the grids of pentode tubes which control the current in the clutch coils of a Lear Magnadrive. This Magnadrive in turn drives the azimuth axis of the Sun-Follower.

Additional stabilization for the azimuth axis was found to be necessary due to the high azimuth inertia and extremely high lock-on rates. This was accomplished by inserting a small condenser across the coarse eye adding resistors, to provide a small additional amount of lead. In this manner it was possible to make the system capable of searching and locking on the sun while traveling at azimuth rates in excess of 720 deg/sec, and following with an accuracy of less than ±0.25 deg. No additional stabilization was required for the elevation servo system due its relatively small inertia. Elevation tracking accuracy also is ±0.25 deg.

The 400 cps power required for the synchronous choppers, and the fixed phase of the 2-phase induction motor in the elevation servo were obtained from a separate oscillator and power amplifier. Anode voltages for the circuitry were obtained from a dynamotor and vacuum tube voltage regulator circuit.

In order to permit conventional electronic circuitry to be used on equipment exposed to very high altitudes, the entire electronic package is placed in a pressurized container. No special precautions are taken to insure that the container is airtight. It is ground tested at 15 psig and sealed so that leakage does not exceed 1 psi per 10 min. As the maximum flight times are less than 5 min, this is considered adequate.

All circuits not pressurized are limited to 27 volts on any exposed contacts.

Telemetry

In order to obtain a continuous check of the operation of the circuitry, various data are telemetered to ground receivers in the vicinity of the rocket launching site. These data are: Plate voltage, 26.5 volt battery supply voltage, Sun-Follower azimuth and elevation pointing angles, azimuth and elevation pointing error signals, azimuth magnadrive clutch currents. An FM-FM telemetering system using a single subcarrier oscillator together with a commutator is used to transmit the required data.

Results of Flight Tests to Date

The history of the previous four Sun-Follower flights show a steady increase in over-all performance and in data gathered by the spectrograph.

The first flight started out with a bang; that is, the V-2 rocket on which the instruments were mounted blew up and was destroyed completely at take-off.

An Aerobee was used to carry the second Sun-Follower. The telemetry records for this flight indicated that the single azimuth sensing eye was pointing away from the sun, although the servo system was nulled. No data were obtained on the spectrograph camera.

An Aerobee was used again to carry the third Sun-Follower. Usable data were obtained on the spectrograph camera, although the servo system was oscillating about the null position. The oscillation apparently reduced the spectrograph exposure time.

A Viking rocket was used to carry the fourth Sun-Follower. Telemetry records indicated that at 120 sec after take-off the rocket began to roll and accelerated to 180 rpm at approximately 140 sec after take-off. This roll rate appreciably exceeded the Sun-Follower design roll rate of 120 rpm. At 200 sec the Sun-Follower finally lost track of the sun because of the high roll rate. There was little usable spectrographic data before 120 sec, due to the atmosphere; however, from 120 to 200 sec the Sun-Follower tracked sufficiently well to obtain the best data of the series so far.

The fifth and sixth Sun-Followers have been built, tested, and delivered by Aircraft Armaments Inc., to the Naval Research Laboratory where they are undergoing installation and check-out tests. The present schedule calls for the fifth and sixth Sun-Follower to be fired on an Aerobee rocket at White Sands Proving Ground about May 22, 1956.

Heat-Up Time of Wire Glow Plugs

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Nomenclature

P = power input, units per unit time, based on the average resistance of the wire over the temperature range to be encountered

A = specific heat of glow plug material $\times$ weight of glow plug

T_w = absolute temperature of glow plug wire at any time

t = time

B = emissivity $\times$ surface area of glow plug wire $\times$ Stefan-Boltzmann constant

T_0 = absolute temperature of chamber walls to which glow plug loses heat by radiation

T^1 = absolute temperature of plug wire at start of heating, i. e., $t = 0$

C = $P + BT_0^4$

γ = $(C/B)^{1/4}$

α = $4B\gamma^3 t/A = 4Ct/A\gamma$

β = T_w/γ

λ = T'/γ

THE time-temperature relationship in a wire resistance glow plug, used as a heat source starter in a reaction chamber, can be obtained with a considerable degree of accuracy by assuming that the primary heat loss of the wire is due to radiation effects. This assumption is justified in many respects since the chamber walls are usually quite close to the wire and almost completely surround it, and since the air remaining inside the system under actual starting conditions is usually stagnant or of negligible velocity. Knowledge of the time-temperature relationship allows the fuel injection to take place at the proper prescheduled time (temperature) after the power input to the glow plug has begun. Alternatively, if the time delay of fuel injection is fixed, solution to the problem enables the proper glow plug to be chosen for a specified set of conditions, such as a desired wire temperature at the time of fuel injection.

Neglecting convection and conduction, the differential equation governing temperature is

$$P = A \frac{dT_w}{dt} + B(T_w^4 - T_0^4) \dots \dots \dots [1]$$

where T_0 is assumed to remain a constant since the chamber mass is almost invariably many times larger than that of the glow plug. Equation [1] may also be written

$$\left(\frac{A}{B}\right)\frac{dT_w}{dt} + T_w^4 = \gamma^4 \dots \dots \dots [1a]$$

The solution of this differential equation can be obtained by direct integration of Equation [1a], written in the form

$$\int_0^t dt = -\left(\frac{A}{B}\right)\int_{T^1}^{T_w} \frac{dt_w}{(T_w^4 - \gamma^4)} \dots \dots [1b]$$

by noting that

$$\left(\frac{1}{T_w^4 - \gamma^4}\right) = \left(\frac{1}{2\gamma^2}\right)\left(\frac{1}{T_w^2 - \gamma^2}\right) - \left(\frac{1}{2\gamma^2}\right)\left(\frac{1}{T_w^2 + \gamma^2}\right) \dots \dots [2]$$

by expansion by partial fractions. The result is

$$t = \frac{A}{4B\gamma^3}\left[2\left(\tan^{-1}\frac{T_w}{\gamma} - \tan^{-1}\frac{T^1}{\gamma}\right) - ln\frac{(T_w - \gamma)(T^1 + \gamma)}{(T_w + \gamma)(T^1 - \gamma)}\right] \dots \dots [3]$$

This may be conveniently reduced to dimensionless form by

noting that the constant γ is the equilibrium or steady state temperature of the wire, since from Equation [1]

$$(T_w)_{\frac{dT_w}{dt}=0} = [P/B + T_0^4]^{1/4} = \gamma \dots \dots \dots [4]$$

Defining the dimensionless ratios

$$\frac{T_w}{\gamma} = \beta$$

$$\frac{T^1}{\gamma} = \lambda \dots \dots \dots \dots \dots [5]$$

$$\frac{4B\gamma^3 t}{A} = \frac{4Ct}{A\gamma} = \alpha$$

and assuming further that $T^1 = T_0$, or that the chamber and plug are at the same temperature at the instant resistance heating of the plug begins, Equation [3] becomes

$$\alpha = 2(\tan^{-1}\beta - \tan^{-1}\lambda) - \ln\frac{(\beta - 1)(\lambda + 1)}{(\beta + 1)(\lambda - 1)} \dots \dots [6]$$

where the limits on the variables λ and β are

$$0 \leq \lambda \leq 1$$
$$\lambda \leq \beta \leq 1$$

Equation [6] may be shown clearly by a plot of α vs. β for various fixed values of λ. This is illustrated in Fig. 1. It is, of course, possible to solve this problem by numerical step-by-step integration, in which case the actual change in resistance with temperature of the glow plug wire may be used, again on a step-by-step basis. Comparison of results obtained by the use of Equation [3], using the average resistance of the wire over the temperature range encountered, with step-by-step calculations shows agreement to within 2 per cent on time to rise to a given temperature.

Fig. 1 α vs. β for fixed λ
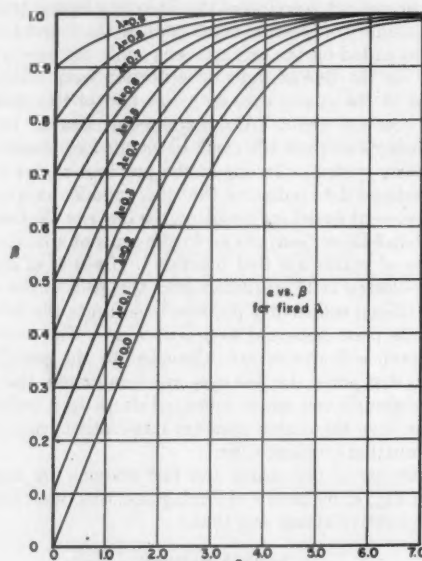
As an example of the use of Fig. 1, assume the following specific wire plug:

Calculated equilibrium temperature = 1800 F
Fixed starting temperature = $T_0 = T^1 = 190$ F
Calculated $4C/A\gamma = 0.5$
To find: Time for wire to reach 1650 F

Calculate:

$$\lambda = 650/2260 = 0.287$$
$$\beta = 2110/2260 = 0.934$$

By interpolation, follow along the $\lambda = 0.287$ curve until reaching $\beta = 0.934$, at which point $\alpha \approx 3.8$. Then

$$\frac{4C}{A\gamma}t = \alpha = 0.5t \approx 3.8$$

$$t \approx 7.6 \text{ sec}$$

The largest uncertainty in the use of Fig. 1 or Equation [3] lies in the choice of the effective emissivity. In regard to this it should be pointed out that a single test in which all other constants are known except emissivity, and in which wire temperature is measured at a specific time before steady-state conditions are reached and again after steady-state conditions are reached, gives sufficient information to determine the average effective emissivity, since

$$B = \frac{A}{4t\gamma^3}\left[2(\tan^{-1}\beta - \tan^{-1}\lambda) - \ln\frac{(\beta - 1)(\lambda + 1)}{(\beta + 1)(\lambda - 1)}\right]$$

Comments on "Manifolds for Solid Propellant Rocket Motors"[1]

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Introduction

IN HIS paper "Manifolds for Solid Propellant Rocket Motors," Mr. Geckler gives a very interesting theory for computing the adequate size of manifold connecting two similar solid propellant rockets. However, the author says he does not know of any experimental tests verifying the validity of his equations.

Many static and flight tests have been conducted by SEPR with manifolds for double-base solid propellant boosters. In static tests, the mechanical and thermal behavior of the manifold was experimented by using nozzles with different throat areas in each rocket motor. Thus, an important mass flow rate of propellant gas was obtained in the manifold. From the results of those tests, experimental values of the factor CY used in Mr. Geckler's theory can be deduced as stated below.

Method for Determining CY

For two rocket motors having the same nozzle throat area A_t, interconnected by a manifold of cross-sectional area A_M, Mr. Geckler writes

$$A_M = A_t \frac{\Gamma(1 - n)(\Delta p_i - \Delta p_f)}{2CY\sqrt{2p_{2f}\Delta p_f}} \quad\ldots\ldots\ldots\ldots [1]$$

where

$$\Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}$$

$\gamma = Cp/Cv$ = ratio of specific heats
n = pressure exponent in the burning rate rule
CY = factor in the mass flow rate rule for the manifold
$\Delta p_i = p_{1i} - p_{2i}$ = pressure difference between the two chambers, without manifold
$\Delta p_f = p_{1f} - p_{2f}$ = pressure difference between the two chambers, with manifold

For two rocket motors having different nozzle throat

areas, A_{t1} and A_{t2} we can substitute for Equation [1] the following equation

$$A_M = \frac{\Gamma(1 - n)[(A_{t1}p_{1i} - A_{t2}p_{2i}) - (A_{t1}p_{1f} - A_{t2}p_{2f})]}{2CY\sqrt{2p_{2f}(p_{1f} - p_{2f})}} \quad\ldots\ldots [2]$$

The factor CY is given by

$$CY = \frac{\Gamma(1 - n)}{2\sqrt{2}} \times \frac{\alpha_1(p_{1i} - p_{1f}) - \alpha_2(p_{2i} - p_{2f})}{\sqrt{p_{2f}(p_{1f} - p_{2f})}} \quad\ldots [3]$$

where

$$\alpha = A_t/A_M \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

The ballistic properties of the double base solid propellant used are known

$$\gamma = 1.26 \qquad \Gamma = 0.66$$
$$n = 0.68 \qquad 1 - n = 0.32$$

Besides

$$p_i = f(A_p/A_t, \theta) \ldots\ldots\ldots\ldots\ldots [5]$$

where

A_p = propellant burning area
θ = propellant temperature

For each test, the different terms of Equation [3] are known

$$\Gamma(1 - n)/2\sqrt{2} = 0.074$$

A_{p1}, A_{p2}, A_{t1}, A_{t2}, and A_M are given; θ is measured; p_{1i} and p_{2i} are deduced from Equation [5]; p_{1f} and p_{2f} are measured; α_1 and α_2 are deduced from Equation [4]; we can then obtain numerical values for CY.

Experimental Results

Experimental results were obtained for values of: $\Delta p_f/p_{2f} = (p_{1f} - p_{2f})/p_{2f}$ varying between 1.5 per cent and 6 per cent.

The precision on numerical values of CY is small because $(p_{1f} - p_{2f})$ is not directly measured, but obtained as the difference between p_{1f} and p_{2f} with an important incertitude. Nevertheless, all the results with the type of manifold represented on Fig. 1 have given values of CY between 0.38 and 0.59

$$0.38 \leq CY \leq 0.59$$
$$CY = 0.48 \pm 0.11$$

Moreover, static tests have shown other advantages of manifolds than pressure equilibrium. Particularly, the simultaneity of ignition is very good (difference inferior to 0.01 s) and so is the simultaneity of extinction.
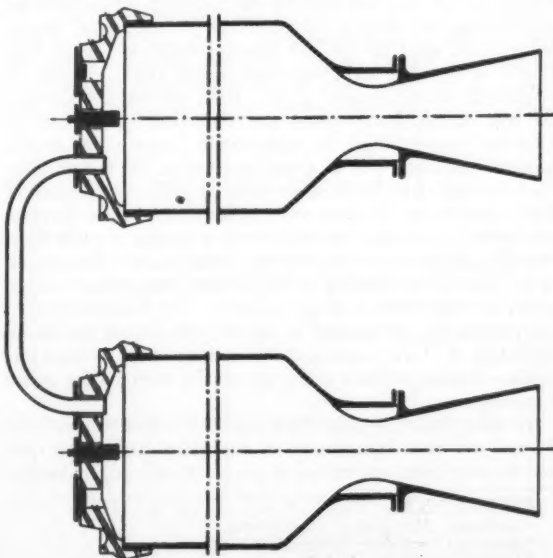
Fig. 1 Type of manifold used on SEPR boosters

Measurement of Total Emissivity of Porous Materials in Use for Transpiration Cooling

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Introduction

TRANSPIRATION or porous-wall cooling is one of the more promising proposals for the maintainence of tolerable temperatures on surfaces subjected to intensive heating. The use of such transpiration cooling has been advocated for cooling combustion chamber walls, gas turbine blades, and external surfaces of high-speed aircraft subjected to aerodynamic heating. The equilibrium temperature of such transpiration-cooled surfaces is determined by a balance between the convective, conductive, and radiative heat transfer. In view of this, the knowledge of the radiative characteristics of porous surfaces is of considerable engineering importance. Such a study is currently under way at the Heat Transfer Laboratory of the University of Minnesota. The purpose of this note is to present a description of the test apparatus that is being used for the determination of the total normal emissivity values of a number of porous materials and to discuss its theory. In addition, total emissivity data are presented for a sintered bronze material. A more comprehensive report covering a number of porous materials will be presented at a later date.

Description of Apparatus

The test arrangement used in the present emissivity measurements is essentially the same as that first described by E. Schmidt (1, 2).[4] It is illustrated in Figs. 1 and 2. The principal piece of equipment is the radiometer which is so designed that the radiant energy entering the opening (a) is collected by a 5-in-diam gold surface mirror (b) and directed to the receiving surface of a thermopile (c) (Kipp and Zonen Microthermopile, E-3). The amount of radiant energy striking the thermopile is controlled by two orifices located in front of the thermopile. The deflection of a sensitive galvanometer attached to the thermopile gives a measure of the energy impingent on the thermopile surface. A black body or ideal radiator (d) constructed of copper and heated electrically serves as a reference for the radiation measurements. The porous test sample (e) is held in a balsa wood frame, balsa being used for its low thermal conductivity of 0.03 Btu/hr ft°F. The porous specimen is held tightly against an electrically heated copper plate. Both the black body and the test sample temperatures are controlled by varying the electrical power input to appropriate heaters. A double walled container (f) is arranged in front of the test sample. Its inner and outer surfaces are painted black with a paint of high absorptivity for long wave radiation and these surfaces are kept at a constant temperature by a stream of water flowing through the space between the double walls. Its purpose is to create a surrounding of well-defined temperature and to avoid any reflection of stray radiation. Its cylindrical opening permits the radiometer to see the test sample and an indentation of $1^1/_4$ in. diam and $4^1/_2$ in. long in the surrounding surface creates a black body (g) at the surrounding guard temperature.

In using this equipment the thermopile is first removed and is replaced by a light source of the same size. The radiometer may then be directed at the black body (d) by looking
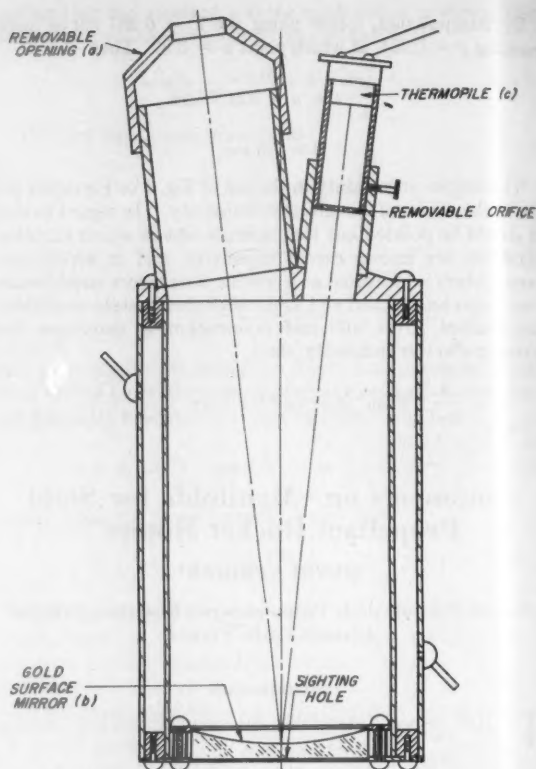
Fig. 1 Emissivity apparatus
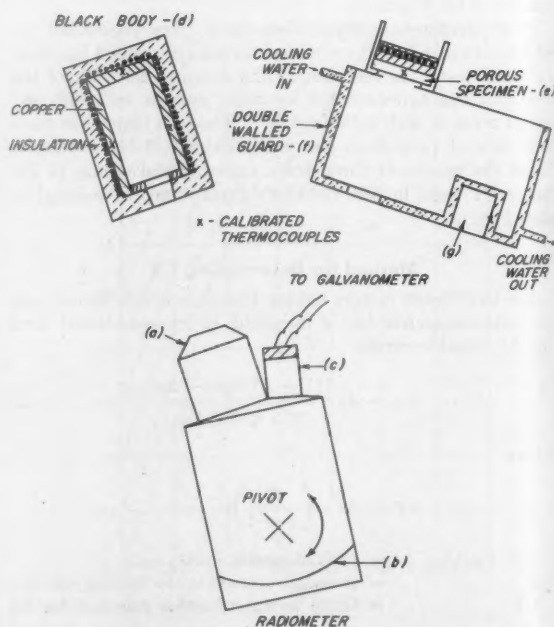
Fig. 2 Schematic of emissivity apparatus

through a sighting hole in the back of the radiometer. The unit is in proper adjustment when the light bundle, after reflection from the gold mirror, goes on through the opening in the black body without hitting the adjacent front walls. Similar checks are made on the sample and on the guard black body (g). The light is then removed and replaced by the thermopile. Then the galvanometer deflections oc-

curring when the radiometer points toward the porous surface (e) and the two black bodies (d) and (g) are recorded. These readings, in conjunction with the corresponding temperature measurements, allow the calculation of the emissivity of the sample.

Analysis of Data

The radiative energy flux leaving the black body at temperature T_b and arriving on a unit area of the thermopile is determined by the solid angle intercepted by the two diaphragms ahead of the thermopile and may be designated as G_1.

$$G_1 = F\sigma T_b^4 \dots \dots \dots \dots \text{[1]}$$

F is an angle factor determined by the geometry and position of the two diaphragms and σ is Boltzmann's constant.

Since the sample at temperature T_s is completely surrounded by a black container at a temperature T_g, the energy flux G_2 leaving its surface and arriving on a unit area of the thermopile is composed of the emitted radiation and the reflected radiation

$$G_2 = F\sigma[\epsilon_s T_s^4 + \rho_s T_g^4] \dots \dots \dots \text{[2]}$$

with ϵ_s indicating the emissivity and ρ_s the reflectivity of the test specimen.

With the assumption $\epsilon_s = 1 - \rho_s$, the following result is obtained from Equation [2]

$$G_2 = F\sigma[\epsilon_s(T_s^4 - T_g^4) - T_g^4] \dots \dots \dots \text{[2a]}$$

The radiant energy flux leaving the small black body in the guard surface at temperature T_g and arriving on a unit area of the thermopile is

$$G_3 = F\sigma T_g^4 \dots \dots \dots \dots \text{[3]}$$

Subtracting Equation [3] from Equations [1] and [2a] and rearranging results in the following expression for the emissivity

$$\epsilon_s = \left(\frac{T_b^4 - T_g^4}{T_s^4 - T_g^4}\right)\left(\frac{G_2 - G_3}{G_1 - G_3}\right) \dots \dots \dots \text{[4]}$$

The assumption $\epsilon_s = 1 - \rho_s$ requires that the sample is sufficiently thick so that none of the impinging radiation leaves it through the back surface and strictly that the emissivity does not depend on wavelength. Equation [2a] is a good approximation even when the last condition is not fulfilled provided T_g is considerably smaller than T_s, or when both temperatures are nearly equal. The possible error on the end result of a variation of the emissivity with wavelengths has to be decided from case to case.

It now remains to relate the radiation fluxes, G, to the thermopile deflection, Δ. A heat balance may be written on the front face of the thermopile for the condition that it is irradiated by energy coming from the black body

$$C_1 G_1 = \sigma\epsilon_t T_{f_1}^4 - q_{rad} + q_{loss} \dots \dots \dots \text{[5]}$$

where

c = loss factor accounting for absorption of energy by CO_2 and water vapor in air and by the gold surface mirror
ϵ_t = emissivity of thermopile
T_{f_1} = temperature of exposed surface of thermopile
q_{rad} = radiant energy, absorbed by thermopile, originating from surrounding walls
q_{loss} = losses from thermopiles by convection and conduction

We may write similar expressions when the thermopile is subjected to radiation from the sample, and from the guard black body.

$$C_2 G_2 = \sigma\epsilon_t T_{f_2}^4 - q_{rad} + q_{loss} \dots \dots \dots \text{[6]}$$

$$C_3 G_3 = \sigma\epsilon_t T_{f_3}^4 - q_{rad} + q_{loss} \dots \dots \dots \text{[7]}$$

If the assumption is made that the decrease in intensity is equal in all cases ($C_1 = C_2 = C_3$), and that the environment radiation to the thermopile and the conduction and convection losses remain the same, we may write, on combining Equations [5], [6], and [7]

$$\frac{G_2 - G_3}{G_1 - G_3} = \frac{T_{f_2}^4 - T_{f_3}^4}{T_{f_1}^4 - T_{f_3}^4} = \frac{4T_{f_3}^3(T_{f_2} - T_{f_3})}{4T_{f_3}^3(T_{f_1} - T_{f_3})} \dots \dots \text{[8]}$$

The error introduced in Equation [8] by writing it in the form on the right is quite small, with the numerator being in error approximately by the amount $1.5(T_{f_1} - T_{f_3})/T_{f_3}$ and the denominator by $1.5(T_{f_1} - T_{f_3})/T_{f_3}$. The value $T_{f_1} - T_{f_3}$ is of the order of 1 to 2 degrees, thereby giving an error of the order of $1/2$ per cent in the numerator and denominator. The error in the quotient is probably less than this amount.

The assumption $C_1 = C_2 = C_3$ may not be admissible in all cases since the absorption of carbon dioxide and water vapor depends strongly on wave length. Provision is, therefore, made to flush the radiometer with a nonabsorbing gas.

The thermopile indication Δ is directly proportional to the differential between the front side temperature, T_f, of the thermopile and the unexposed back side temperature, T^*.

Therefore, when the radiometer is sighted on the black body, the indication is

$$\Delta_1 = k(T_{f_1} - T^*) \dots \dots \dots \text{[9]}$$

The thermopile is so constructed that T^* remains constant when the radiometer is directed toward the other surfaces, yielding the following relations

$$\Delta_2 = k(T_{f_2} - T^*) \dots \dots \dots \text{[10]}$$

$$\Delta_3 = k(T_{f_3} - T^*) \dots \dots \dots \text{[11]}$$

Substituting Equations [9], [10], and [11] in Equation [8], and combining with Equation [4], we arrive at the final relationship for ϵ_s

$$\epsilon_s = \left(\frac{T_b^4 - T_g^4}{T_s^4 - T_g^4}\right)\left(\frac{\Delta_2 - \Delta_3}{\Delta_1 - \Delta_3}\right) \dots \dots \dots \text{[12]}$$

Thus the knowledge of the three temperatures and the deflections $\Delta_2 - \Delta_3$ and $\Delta_1 - \Delta_3$ allows the calculation of the emissivity of the porous sample.

Total Emissivity Values for Porous Bronze

Data were obtained for a sintered bronze material currently used for transpiration cooling experiments. The test sample of 4 in. sq was removed from a large plate 14 in. wide and 72 in. long $\times$ $1/2$ in. thick. Its porosity is such that an 8 psi differential across the plate allows an air flow of 0.55 lb per sq ft per sec. It was fabricated by Powdercraft, of Spartanburg, South Carolina. The surface roughness of the plate
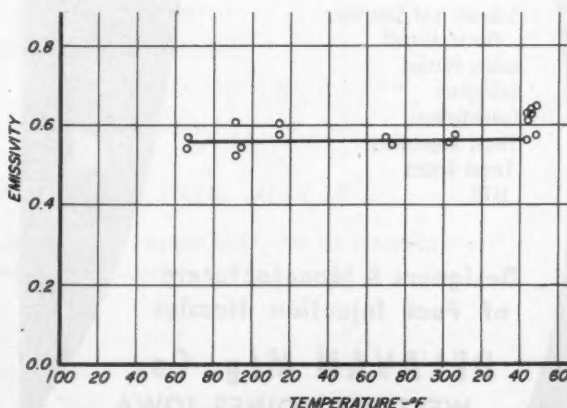
Fig. 3 Total normal emissivity of porous bronze

was found to have an absolute average roughness value of 0.0005 in. when it was measured with a Brush Surfindicator having a stylus with a tip radius of 0.0005 in.

The test results are presented in Fig. 3 and cover a temperature range of 160 F to 350 F. The emissivity value found for this material is 0.57 ± 0.04. At 350 F the sintered bronze rapidly oxidized which resulted in an increase in the emissivity value to 0.63.

The emissivity value of the solid bronze surface obtained by grinding the porous test specimen to close the openings was measured and found to be 0.04 at 150 F. This is an order of magnitude lower than the value for the porous surface and emphasizes the possible error involved if emissivity values for solid surfaces are used to predict the radiative heat transfer from porous materials.

Acknowledgments

The authors acknowledge the able assistance of R. Eichhorn and Richard Birkebak in the design of the radiometer and, in addition, wish to express appreciation to F. J. Bradac for his skillful construction of the radiometer.

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The Near-Constancy of Full-Power Duration for Unboosted Rocket Vehicles

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THE firing duration (at constant thrust) of a rocket engine which constitutes the sole propulsion system of a piloted craft or single-stage missile may conveniently be expressed in terms of the initial thrust-weight ratio F/Wo, the mass ratio ζ, and the specific impulse I_{sp}

$$T = I_{sp} \zeta \, Wo/F \dots\dots\dots\dots\dots [1]$$

One example of magnitudes, for a piloted aircraft, might be $I_{sp} = 220$ sec, $\zeta = 0.5$, $F/Wo = 1.0$, for which combination $T = 110$ sec. Thus, a rocket-propelled fighter of moderately high performance might operate less than two minutes *at full throttle.*

A second typical set of magnitudes for insertion into Equation [1] would apply to a missile of advanced design where $I_{sp} = 250$, $\zeta = 0.9$, $F/Wo = 3.0$; giving a duration $T = 75$ sec. This vehicle would, if launched vertically, reach a speed of the order of Mach 2.5 at 15,000-ft altitude and an acceleration of 30 g's at burnout. These conditions might be intolerable because of drag and stress, respectively, and a 25 per cent reduction of design thrust ($F/Wo = 2.25$) would result in $T = 100$ sec.

It is rather striking to find this latter figure so near to the full-throttle duration of the piloted aircraft considered in the first example. This near-constancy of full-powered operating time for unboosted vehicles of widely varying designs has been recognized for some time; however, the elementary Equation [1] may be an aid to quick visualization of the influence of basic design parameters in this regard.

Power plant design questions of a most fundamental sort, such as pressurization and cooling, depend upon firing duration. Because of the near-constancy of duration often found, these questions may have unexpectedly similar answers for widely different applications.

Jet Propulsion News

Alfred J. Zaehringer, American Rocket Company, *Associate Editor*

Norman L. Baker, Indiana Technical College, *Contributor*

Jet Aircraft, Engines

TRANSPORTS. New version of the Boeing 707 jet transport is called the Intercontinental (*photo*). The plane will gross 290,000 lb and will carry 124 or 145 passengers. The range will be over 4000 miles. Power will come from four P&W JT4 turbojets. Specifications include: length, 146 ft, 8 in; span $141\frac{1}{2}$ ft; payload will be 36,500 lb; fuel capacity is 21,200 gal. All models will be equipped with the new Boeing-developed combined sound suppressor and engine thrust reversers. Pan American World Airways has ordered 12 jets for delivery in August, 1959. Other orders include: 4 for Continental Air Lines; Air France, 10; Sabena Belgian World Airlines, 3; and 5 for Braniff Airways.

Intercontinental jet transport

Boeing

● Orders for the Douglas DC-8 jet are also rolling in. Eastern Airlines has ordered 26, Scandinavian Airlines 7. Orders now total 95. Douglas has stated that although the J-57 turbojet is presently provided for in the DC-8, use of the Rolls-Royce Conway bypass engine is also under consideration for foreign use.

● Other jet transports have also been making news. The Comet 3 on a round-the-world flight is expected to cover 30,000 miles in a flight time of about 68 hours. The Comet 3 is a prototype of the Comet 4 which goes into public service in 1959. Comet 4 will carry 60–76 passengers over stage lengths of 2700–3000 miles. Optimum cruising speed is Mach 0.74 (about 500 mph). Maximum takeoff weight will be around 152,000 lb.

● Deliveries of the Britannia, Britain's latest turboprop transport, will be made in 1957 to Canadian Pacific Airlines. The Britannia carries 100 passengers at 400 mph and at ranges up to 6000 miles.

● Douglas Aircraft Co. has announced that 25 DC-8 jet airliners were sold to Pan American World Airways. The Pan Am purchases, first for the DC-8, were in excess of $160 million. Deliveries are scheduled for 1959. Another order for 30 DC-8's was placed by United Airlines. Deliveries are also scheduled in 1959. It is expected that flight times in the U. S. will be materially reduced. For example, the Chicago–New York run will be around $1\frac{1}{2}$ hr (about 1 hr less than best existing times) while the Los Angeles–New York trip will be around $4\frac{1}{2}$ hr (about 3 hr better than existing runs).

● The Air Force has ordered the Convair B-58 supersonic bomber into initial production and the building stage is near at Forth Worth, Texas.

JET ENGINES. The 500th gas turbine engine was completed by Boeing. The engine was one of the 502-7 series. The lastest production engines weigh 320 lb complete with accessories and oil. It produces 270 hp maximum and 240 hp on a continuous basis. Specific fuel consumption is about 1 lb/hp/hr. Time between overhauls will be about 1000 hours. An automobile equipped with a similar gas turbine engine, for example, would travel at over 100 mph and would travel over 100,000 miles without engine overhaul. It is anticipated that by 1957, the fuel consumption of these engines can be expected to drop to about 0.7 lb/hp/hr and making it competitive with gasoline engines of the same power output. A 175-hp engine was successfully used to propel a 24-ft Navy personnel boat (*photo*).

Gas turbine afloat

Boeing

● Precision forging of titanium jet turbine blades was announced by the Canadian Steel Improvement, Ltd. Secret of the process lies in protecting the titanium during forging and heat treatment. The process was developed for Orenda Engines in Malton, Ontario. The blades are to be used on turbojets now being produced for the Avro CF-100.

deHavilland

Static test of **SUPER SPRITE**

FIGHTERS. New drag chutes were recently installed on the F-84F jet planes and have given landings in only 2400 ft. This is 1000 ft shorter than the normal landing without chute. The chute is 16 ft in diameter and trails 35 ft behind the craft. At 130 knots, the drag of the chute is about 8000 lb. The equipment is being installed on present production equipment.

● First flight of the F-105A, the USAF-Republic supersonic fighter-bomber has been completed at Edwards AFB, Calif. The model is powered by a P&W J-57 turbojet engine of 10,000-lb thrust. The craft has short, very thin swept wings, a long cylindrical fuselage, wing-root air scoops, and a one-piece flying tail that sets low on the aft fuselage section. Unusual feature of the ship is a vertical fin on the bottom aft fuselage section for lateral stability. Classed as a weapons system, the F-105A can carry nuclear weapons, as well as conventional bombs and rockets.

● Crusader, the F8U-1 jet fighter by Chance Vought is to be produced under a $100 million production order. The carried based Navy fighter has flown supersonically in level flight and gets its thrust from a P&W J-57-P-12.

● Two light fighters have been in the news. The British Folland Gnat has hit close to Mach 0.95 in flight tests. Ceiling of the small 7000 lb craft is above 50,000 ft. Powered by a Bristol Orpheus turbojet of about 4000 lb thrust, specs are: span, 22 ft, 2 in; length, 29 ft, 9 in. The French Dassault Mystere 22 is a NATO fighter grossing 12,000 lb. With a 45-deg sweepback, the craft is powered by two Turbomeca jets of 2200 lb thrust each. Armament is expected to be 30-mm cannon, rockets, and external stores.

● The Soviets have disclosed that their twin-jet medium bomber has a range of about 3000 miles. The bomber has also been modified for use as an airliner.

Facilities

● DuMont has consolidated its missile work in a new Missiles Engineering Department. Los Angeles may be the site for this group.

● Sperry Rand Corp. is locating its new Air Division in Phoenix, Ariz., and will include work on drones and pilotless aircraft.

● Steel RATO sub-assemblies will be made by the Norris-Thermador Corp. in a new $2 million plant at Waco, Tex. Work will be concerned with the new T-60 RATO being produced by Phillips Petroleum at McGregor, Tex.

● Philadelphia has been selected as the new headquarters of the Special Defense Projects Department of General Electric. The department is currently engaged in research and development work on several large weapons systems in the guided missile field. The move from Schenectady will take place in mid-1956 and total employment will be about 1000 personnel.

● Rocketdyne is the name of the rocket division of North America Aviation. Headquarters of the $9.5 million establishment is taking shape at Canoga Park in the San Fernando Valley of Calif. Rocketdyne will be responsible for the design, development, manufacture, and tests of rocket power plants for aircraft and missiles and for the design and development of small caliber aircraft rockets. The division supplies the ATO rocket for SM-64 NAVAHO and rocket power plants for ATLAS and REDSTONE. Downey Division is now working on SM-64 (*photo*). All large rockets, however, are static tested at the Simi Hills Field Test Laboratory in the Santa Susana Mountains.

● Two new supersonic wind tunnels were announced by industry. Republic Aviation is to build a Mach 4 tunnel costing $12 million at its Farmingdale, N. Y., facility. Boeing is putting the finishing touches on its 9 × 9-in. tunnel at Wichita, Kan. This tunnel is also of Mach 4 design (*photo*).
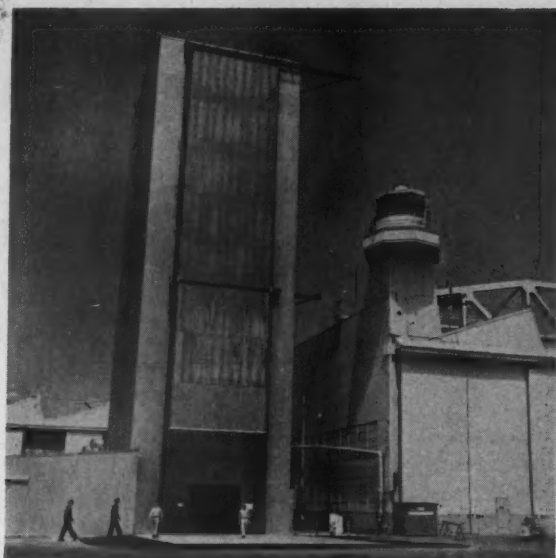
North American

Simulation tower for flight loads on NAVAHO
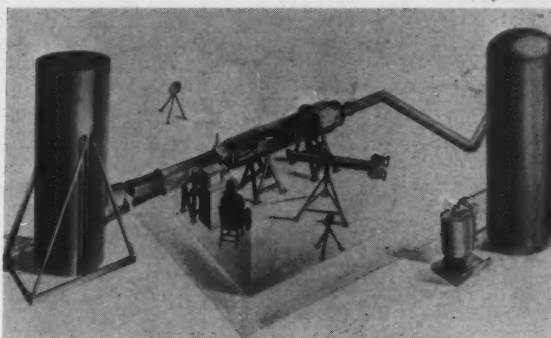
Boeing

Supersonic wind tunnel. Compressed air stored in tower at left passes instrumented test section (center) and expanded and discharged through tower at right

ARS Regular Feature on Detroit T. V.

"Rocket Digest" is a regular Friday night ARS program on Detroit Station WTVS. Featured on the show is Alfred J. Zaehringer (left), Associate Editor of JET PROPULSION. Recently a live firing was made of one of Zaehringer's medium size gas generator units—the smokeless, odorless, variety. The half-hour program is moderated by ARS member Donald J. Kenney (right), a member of the faculty at the University of Detroit.
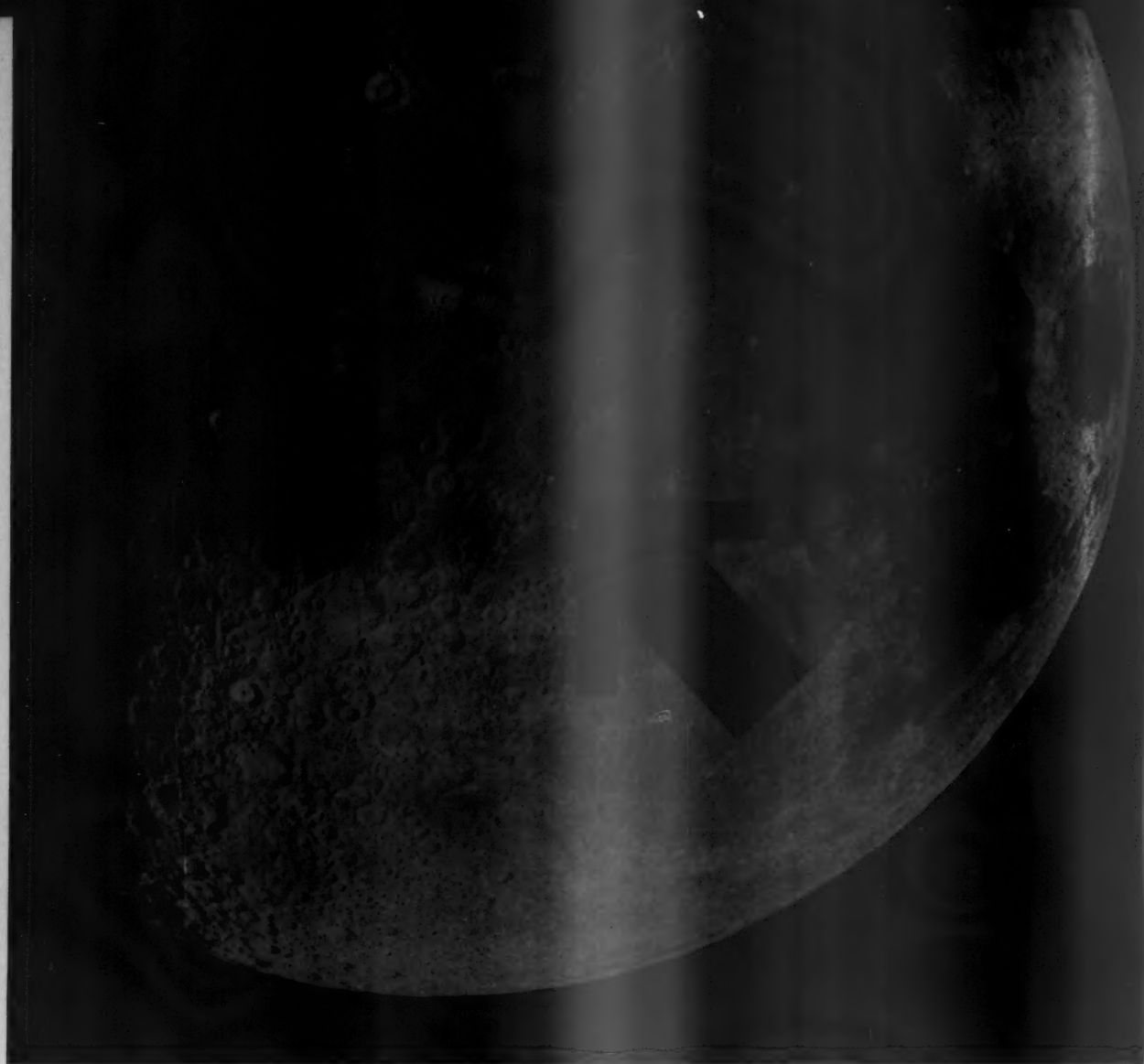
NEW FRONTIER

"If we had to put a man on the moon, we could do it."
—Overheard at an Institute of Aeronautical Sciences luncheon

This impromptu statement was not a matter of idle conjecture. It was a statement of a positive and scientific fact – as provable as if he'd said the Aleutian Islands – and contingent only upon three prime requisites: enough time, money, and necessity. And by "we" he meant today's mindpower and facilities operating under the most advanced concepts of research and development.

Those concepts as practiced at Martin today would be essential to the fastest possible solution of any complex flight systems problem now within the capacity of man to solve.

It is this that has established Martin as one of the prime forces in the coming conquest of the new frontier—Space itself.

MARTIN
BALTIMORE

Modern Falconry
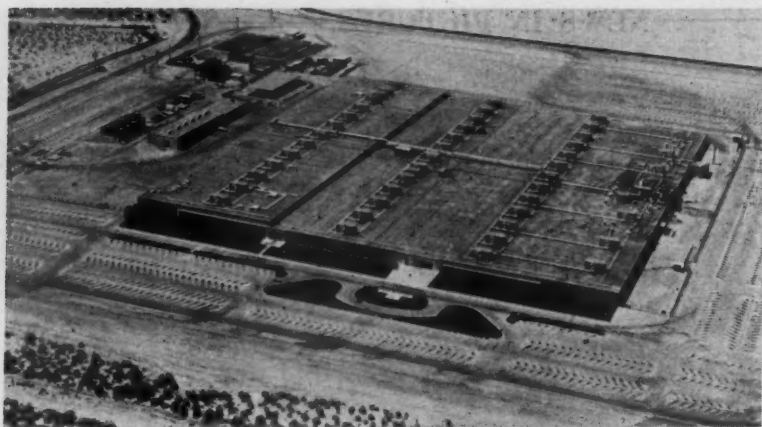
Falcon Nest...Home of the FALCON is at the Tucson, Arizona, plant of Hughes Aircraft Co. Designed for the GAR-1 production, the plant, with 12 acres under one roof, was constructed in 1952 at a cost of $8 million. It contains over $6 million of production equipment and $15 million of special test equipment
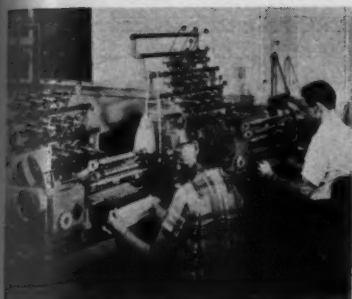
Hughes

Missile Bobbins...FALCON's "innards" are fashioned on these multiple-coil winding machines each winding 12 coils of copper wire. Coils will be used in electromagnets for control system. Entire plant is air conditioned at higher than atmospheric pressure to guard against dust
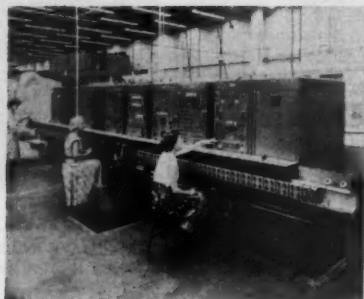
Hughes

Tube Tester...Capable of testing over 600 vacuum tubes per hour, this automatic machine is loaded at left. Shock and vibration are among some of the tests simulated

Hughes

Shaping Up...Forward sections, magnesium shells, begin to take shape on this sheet metal assembly line
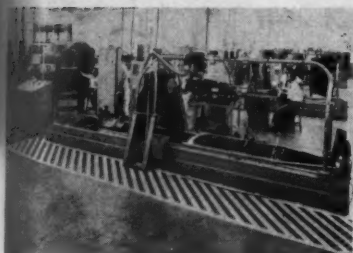
Hughes

Cradled...FALCON is about to undergo vibration tests suspended in this cradle. Shock tests are conducted in the bottom of the cradle. Note altitude test chamber in background

Hughes

(*Left*)   *Hughes*

Bird in Hand...Smallest guided missile in production, the GAR-1 tips the scale at over 100 lb. Power comes from a Thiokol solid propellant rocket motor. Although having a typical canard layout with small forward fins and large aft supporting surfaces, the forward wings are fixed and steering is accomplished by control surfaces hinged to the trailing edge of the stabilizers. A round nose fairs into a cylindrical fuselage which terminates in a boat-tail housing rocket exhaust nozzle

Missile Row...Final assembly and testing completed, the birds are boxed for shipment to the Air Force

Hughes
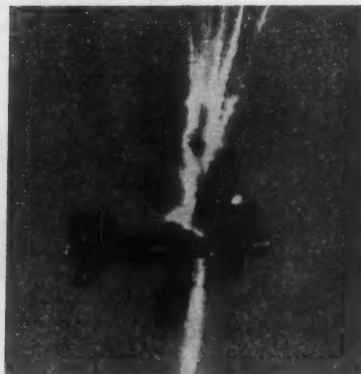
Falcon Strikes...Makes direct hit on target drone aircraft in tests

Thiokol
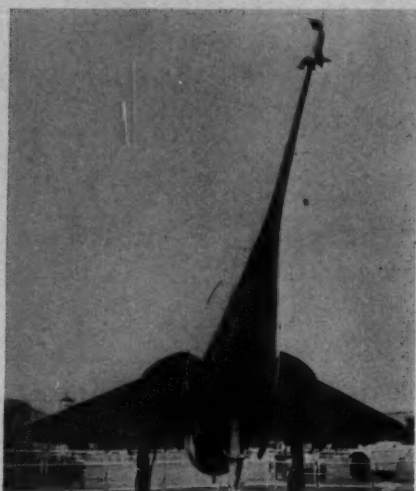
British Information Services

DROOP-SNOOT. . .Second British aircraft capable of supersonic speed in level flight is this Fairey Delta 2, single-seat craft with an elongated nose that can be lowered like a drawbridge to give pilot better vision for landing, take-off, and taxiing. Wing span is 26 ft, 10 in; length is 52 ft, 3 in.
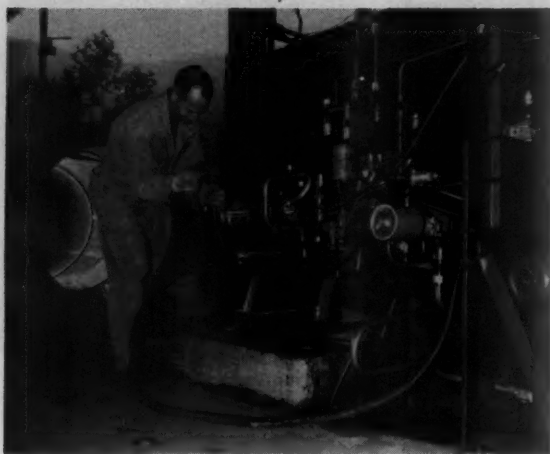
JPL

PLUMBERS NIGHTMARE. . .Technician finishes hooking up a liquid rocket motor for test. Note observation window in center, armor plate at right protecting vital components, exhaust silencer at left
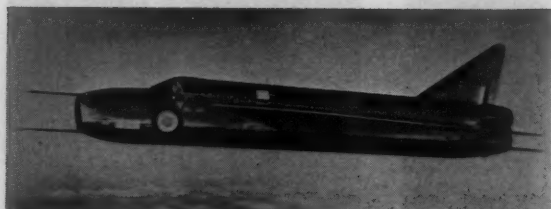
British Information Services

BUGLIKE. . . English Electric P-1 interceptor was the first British plane to hit sonic speed in level flight. It has been flying since 1954 and has made over 100 flights. 20 preproduction models have been ordered by the RAF

British Information Services

GROUND LEVEL EJECTION. . .British ejection seat has successfully worked at ground level and speeds of 130 mph. Top, dummy is fired from aircraft. Bottom, it parachutes to safety

University of Michigan

HOT CAN. . .Observers watch progress on J-47 turbojet burner in test stand

Aerojet-General

AND AWAY WE GO. . .PV-1 airplane equipped with internally mounted rocket engines makes take-off. Two 15KS-1000 units are being used here

ARS News

One Thousand Attend Los Angeles Conference on Thermal Barrier, Satellites

NINETEEN SESSIONS—including one ARS session which was conducted twice in order to accommodate demand—featured the March 14-16 ARS-ASME Aviation Division Conference at the Hotel Statler, Los Angeles. Cosponsors were the ASME Instruments and Regulators Division and the Institute of the Aeronautical Sciences.

Two sessions were run under ARS sponsorship. One of them, on Earth Satellites, attracted a total of some 600 people in two repeat sessions.

David Shonerd of Aerophysics Development Corporation was ARS chairman of the meeting. The ASME Aviation Division Chairman, George T. Hayes, Stanford Research Institute, announced that total registration approached 1000.

The Aviation Division offered sessions on Air Cargo (market analysis, commercial applications, military research forecast, military applications); on the Thermal Thicket (human factors, structural materials, optimum airframe structures, accessory drives, factor of safety concepts, propulsion, controls) and the Instruments and Regulators Division on aircraft instrumentation.

Two luncheons were held and the featured speaker at the banquet was C. C. Furnas, Assistant Secretary of Defense for Research and Development. Dr. Furnas, Fellow Member of ARS, stressed the importance of keeping the public well informed on the nation's defense setup and of maintaining U. S. leadership in research.

Summary of ARS papers:
EARTH SATELLITES

Project Vanguard—The IGY Earth Satellite (289-56), F. R. Furth, Farnsworth Electronics Company. Successful construction and launching of the satellite will demand going to the present limits—or beyond—our engineering knowledge in many fields. Admiral Furth discusses the problems involved, the techniques to be followed, and the roles to be assigned to various military and scientific organizations concerned with implementing the three-year project.

Scientific Aspects of the Satellite Program (288-56), W. H. Pickering, Jet Propulsion Laboratory, California Institute of Technology. Observations made in the field of upper-atmosphere research are briefly reviewed, followed by a discussion of the actual scientific observations which can be taken from the earth satellite. Many experiments can be performed with no instruments aboard, but other experiments require carrying instruments of a type in which the data can be sent to the earth by radio. First flights will certainly carry experiments to measure the kind of environment existing on the satellite because this factor will affect the operation of instruments.

HIGH TEMPERATURE MATERIALS

Some Aspects in the Development and Performance of Pure Oxide Coating: (Section 1), R. V. Westerholm and W. M. Wheildon, Norton Co.; and **Application of Rokide A Coating to Ramjet Engine**

Combustion Components (Section II) (295-56), Alan V. Levy, Marquardt Aircraft Company. Designers of rocket propulsion units and high performance aircraft turbines are approaching the limitations of existing materials. Section 1 of this paper is limited to the broad line of approach that if we discount design improvement (such as various cooling methods) we can employ existing materials to their maximum low temperature physical properties by providing adequate surface protection and insulation by means of a coating.

Section II discusses flame-sprayed alumina coating as an insulating refractory coating for ramjet applications.

High Temperature—Short Lived Plastic Laminates (292-56), Gerald E. Dodson, H. I. Thompson Fiberglass Co. New design concepts and higher performance requirements have shown the need for developing plastic laminates capable of withstanding extremely high temperatures (2000 to 5000 F). Temperature span includes the melting points of most metals and is far beyond the long time temperature resistance of any plastic material (including the irradiated materials). There are three families of plastic resins in use that are suitable both costwise and reproducibility-wise for use in the high temperature ranges. These resin families are discussed in detail together with general manufacturing techniques designed to utilize the best physical characteristics of the laminates.

Is There a Thermal Barrier for Supersonic Ramjet Tailpipes? (293-56), Walter Unterberg, Marquardt Aircraft Company, Single-walled metal tailpipes, internally ceramic-coated where necessary, encounter

Stapp Delayed by Weather, Truax Pinch Hits in Wichita.   Attendance: 660

(Wichita Eagle)

HE PINCH HIT

Commander Robert C. Truax, national Vice President of ARS, was slated only to present the charter to the newly formed Wichita Section on Jan. 27. On his arrival, however, he found that the scheduled speaker, Lt. Col. John P. Stapp, renowned aeromedical scientist, had been delayed by weather, and 460 dinner guests were present to hear him speak. Commander Truax, again demonstrating the stuff of which old time rocket men are made, went "on" and regaled his audience with the "old days" in the liquid propellant field. He is shown above *(third from left)* with John P. Gaty, Vice President of Beech Aircraft; James F. Reagan, Beech's manager of missile engi-

HE WENT ON ANYWAY

neering and President of the Section; and Brig. Gen. L. C. Coddington, Commander of McConnell Air Force Base.

On the following day, Col. Stapp did arrive and 200 "second nighters" appeared at McConnell Air Force Base to hear his lecture on his rocket sled experiments. Above photo shows Col. Stapp *(left)* with Lawrence J. McMurtrey, power plant staff engineer at Boeing Airplane Co., and Vice President of the Section; and Dr. Reagan. They are inspecting a model space vehicle designed and built by David Henderson, an aeronautical engineering student at the University of Wichita and member of the Section's board of directors.

a thermal barrier only in the high Mach number-low altitude region where more complex cooling means are required. A high-performance variable-geometry "paper" ramjet was chosen as example, with internal gases at 4000 F throughout, near the maximum obtainable from hydrocarbon-air combustion. Graphs show how the tailpipe metal temperature at thermally critical locations is affected by the major variables of steady state flight conditions, ground and air launch transients, tail-pipe exit geometry, wall thickness, and wall material.

New Mexico–West Texas Section Establishes Scholarship

TO stimulate interest in rocketry and provide assistance to a qualifying junior student during his senior year at New Mexico College of Agriculture and Mechanic Arts, a $200 scholarship will be given by the New Mexico–West Texas Section. The award will be made at the June Commencement for the best paper on a subject related to the field of rocketry. Papers must be submitted for certification of standing and classification through the dean of the appropriate school and received by the Section by May 1, 1956.

More Student Chapters Forming

FOUR more student groups have submitted by-laws and petitions to become chartered as student chapters of ARS. The University of Michigan student chapter was first to be chartered, on December 8, 1955.

The new units and their officers are as follows:

Academy of Aeronautics, *Flushing, N. Y.*

President, Alex Havriliak

Vice President-Treasurer, Anthony Varvaro

Secretary, Norman Lange

Faculty Advisor, Ralph Hautau

New York University, *Bronx, N. Y.*

President, Ned Greenberg

Vice President, Bertram Kramer

Secretary, Larry Gottlieb

Treasurer, Andrew Dapuzzo

Faculty Advisor, Michael Maccarone

Polytechnic Institute of Brooklyn, *Brooklyn, N. Y.*

President, Donald Morin '57

Vice President, Mario Cardullo '57

Secretary, Bernard Shatz '57

Treasurer, J. Codispoti '58

Faculty Sponsor, T. P. Torda, Department of Mechanical Engineering

Under the guidance of Professor Torda, the group is developing a liquid-fueled motor. Project coordinator is William Peschke. Design group members include R. Mootchnik (injector systems), Morin (chamber configurations), I. Reba and F. Schuyler (exhaust nozzles), Cardullo (propellants) and Peschke (feed systems). It will have a pressure feed system and is being designed for 250 lb thrust.

Charter Presented to North Texas Section

George P. Sutton of North American's Rocketdyne Div., national director and membership chairman of ARS, presented the charter for the newly formed North Texas Section on Feb. 17. Accepting the charter on behalf of the Section was George H. Craig of Convair, Fort Worth, president.

Shown at the ceremony are (*left to right*) Jim Nolan, Convair, program chairman; John Kerr, Convair, director-at-large; Jack Wurts, Chance Vought, secretary; Sutton; Harry Graham, Chance Vought, vice president; and Craig. Charles F. Crabtree, Convair, treasurer, was not present when picture was taken.

Cardullo is editing a newsletter for the group, entitled "Rocket Study."

On February 24 a meeting was addressed by C. C. Miesse of Aerojet-General's liquid engine division on combustion instability.

Saint Louis University, (Parks College), *East Saint Louis, Ill.*

President, James M. Llewellyn, Jr.

Vice President, Dale Anderson

Secretary, Frank Brown

Treasurer, Joe Kelly

Technical Activities Chairman, Pete Butkewicz

Faculty Sponsor, Franz Hug

Donald J. Morin, President, Polytechnic Institute of Brooklyn Chapter

Niagara Frontier Elects Smith of Bell

WILLIAM M. SMITH, chief rocket engineer at Bell Aircraft Corp., Buffalo, has been elected president of the Niagara Frontier Section for 1956. He succeeds Tommie Zannes, also of Bell.

Following are the other officers named: Stanley F. Lewinski, Bell, vice president;

John H. Keefe, Becco Chemical Div., Food Machinery & Chem. Corp., treasurer; Daniel Y. Sing, Bell, treasurer; Walter J. Zebrowski, Convair, recording secretary.

Directors include, for two-year terms: Harry Ferullo, John van Lonkhuyzen, and Zannes, all of Bell; and D. E. Morrison, Orenda Engines, Ltd. Serving for one-year terms will be H. O. Kauffman, Becco; John Blanton, Bell; George Markstein, Cornell Aeronautical Laboratory; and Delbert D. Thomas, Olin-Mathieson Chemical Corp.

Section Doings

Arizona. A March 7 meeting, at the Tucson residence of C. J. Green, president, featured James J. Harford, ARS Executive Secretary, who discussed the Society's history and future plans. The Section members also talked over plans to launch an educational program on rockets in local high schools, as well as a Section publication.

California Desert. A meeting at the Naval Ordnance Test Station, China Lake, was attended by some 66 people. Heinz Haber spoke on the subject of Space Medicine, followed by a tour of the station. Richard Schmidt, Assistant Chief, Rocket Engine Test Station, Edwards AFB, is in charge of organization of this Section, and prospective members in this area may obtain application forms by writing him.

Central Texas. James J. Harford, ARS executive secretary, visited the Section on March 6. He toured the plant of the Phillips Petroleum Co., Rocket Fuels Div., in McGregor, and had an informal dinner meeting with Section officers and directors Elton E. Rush, Fred Sauls, A. C. Keathley, and John Alden, all of Phillips.

Chicago. Local Sections of the IAS

and ARS joined forces for the first time at the February meeting at Illinois Institute of Technology. A pre-meeting dinner was held at the IIT Student Union. Speaker was Emerson W. Smith of North American Aviation, who discussed the practical aspects of aircraft power plant installation testing. Several charts and photos were used to illustrate his talk.

Columbus. A brief discussion on proposed by-laws preceded the February meeting addressed by A. N. Tifford, Aeronautical Engineering Dept., Ohio State Univ. His lecture on escape from earth was followed by a motion picture on rockets.

Indiana. S. V. Gunn of the Rocketdyne Div., North American Aviation, Inc., spoke to approximately 75 persons at the February meeting. Dr. Gunn's subject included a discussion of problems associated with the development of large thrust rocket engines. He also discussed the activities of the newly formed Rocketdyne Div., and briefly described the test facilities located at the Santa Susana Test Station.

Maryland. Sixty members and guests attended the February meeting held at Johns Hopkins University. Guest speaker was Kurt Stehling whose subject was "Winged Rockets." He showed slides and films of the Bell X-1A and X-2 research aircraft.

New Mexico-West Texas. Invitations in the form of perforated strips of "Space tour" tickets were issued by the Section in connection with their "Annual Solar System Soirée" in March. The tickets were "good for passage for two" from the White Sands Proving Ground Navy Officers BOQ, stops at a space station and six planets, and return to Earth. Each portion of the ticket featured an appropriate symbol of the planet named. "Inflight" punch was served to the "space travelers."
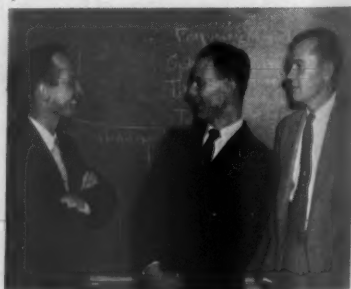
Commander L. I. Stecher *(center)* who addressed the New Mexico-West Texas Section recently on "The Inter-Range Instrumentation Group." *Left*, program chairman R. Gilbert Moore, and *right*, section president R. K. Sherburne.

New York. At the February 10th meeting at IBM World Headquarters, Gilbert Parker of IBM discussed the field of scientific computing with specific emphasis upon large electronic computing equipment. A film was shown ("Piercing the Unknown") in which the use of computing equipment was demonstrated in modern aeronautical and rocket applications. Mr. Parker discussed general types of modern computers and how they can assist the engineer and scientist in

calculating problems associated with rock-
etry. The talk was followed by a demon-
stration of computers operated at the
data processing center.

Southern California. Homer E. Newell,
Jr., acting superintendent of the Atmos-
phere and Astrophysics Division of the
Naval Research Laboratory, was the
speaker at the February meeting in
Pasadena. His subject was the use of
rockets and satellites for geophysical re-
search. Dr. Newell has specialized in
upper atmosphere research at NRL for
nearly ten years.

Wichita. About 100 members and
guests attended a March 1 meeting at the
Beech Employees Club which featured
the appearance of James J. Harford,
ARS executive secretary, as speaker.
A film program followed.

Five Companies Join ARS

TWO Michigan organizations, two from
California, and one eastern company
are recent applicants for Corporate
Membership in ARS. The firms, their
fields of interest, and their Corporate
representatives are:

Accessory Products Corporation, Whit-
tier, California. Valves, regulators, fil-
ters and allied equipment for control of
fluids. Research on high flow fuel, high
and low pressure pneumatics and environ-
mental test equipment for aircraft, rockets
and missiles. Robert G. Rogers, Presi-
dent; Robert L. Maple, Executive Vice
President; William M. Smith, Vice
President, Chief Engineer; Orville A.
Cordes, Sales Manager; William P.
Taylor, Project Engineer.

*Chrysler Corporation—Missile Opera-
tions,* Detroit, Mich. Producer of U. S.
Army's Redstone Missile. Aerodynamics,
ballistics, guidance and control, handling,
launching, propulsion of missiles. C. A.
Brady, Operating Manager; J. P. Butter-
field, Chief Engineer; B. J. Meldrum,
Business Manager; C. W. Williams,
Manager, Fabrication and Assembly;
R. M. Graham, Manager, Quality Control.

Consolidated Western Steel Division,
United States Steel Corporation, Los
Angeles, Calif. Maker of motor cases for
various rocket and missile programs.
Charles E. Fife, contracting Engineer;
Charles R. Fenninger, Contracting Engi-
neer; J. Greer Thompson, Assistant Vice
President, Sales; George E. Boyd, Con-
tracting Manager, General Sales; M. P.
Klick, Chief Engineer.

Electro-Mechanical Products Company,
Garden City, Mich. Manufacturer of
guided missile components, flight simu-
lators, propulsion controls, optics, and
varied electro-mechanical devices. Jack
M. Beauchamp, President; George T.
Spencer, Sales Manager; Chester C.
Quantz, Chief Engineer; Wallace H.
Franklin, Staff Engineer; Frank W.
Beauchamp, Executive Vice President.

Norden-Ketay Corporation, New York,
N. Y. Airborne automatic control sys-
tems and components. G. D. Butler,
Director of Sales; B. Levine, Director of
Design and Application Engineering;
C. F. Schaeffer, Technical Director,
Norden Laboratories Division; R. Sey-
mour, Chief Engineer, Instrument and
Systems Division; R. Porter, Chief
Engineer, Precision Components Division.

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*near Binghamton and Endicott, N.Y.

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equipment required to take over the complete job. The highest precision standards are maintained in engineering, manufacture, quality control, packaging and accounting.

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Electrical, Electronic

Missile Alternator. For short-time operation, weighs 15 lb; 400 cycle, 12,000 rpm. Can be rated to 2000 va., 3-phase 115 v. Specialty Component Motor Div., General Electric Co., Schenectady 5, N. Y.

Line Pressure Transducer. Rated at 250 psig at differential pressures up to 150 psid. Statham Laboratories, 12401 W. Olympic Blvd., Los Angeles 64, Calif.

Missile Heating Element. Neoprene coated, can be used as rocket-tube heater where only moderate heat is required. Higher temperatures available. Safeway Heat Elements, Inc., Middletown, Conn.

Direct-Reading Pre-Amplifier. Model 2614 is a probe type for piezoelectric pickups. With VTVM can read acceleration, pressure, force. Endevco Corp., 180 E. California St., Pasadena 1, Calif.

Teledyne Strain Gage. New pressure transducer, 2¹/₄-in. diam and 3¹/₂ in. long available in eight ranges up to 10,000 psi. Taber Instrument Corp., Section 36, 111 Goundry St., N. Tonawanda, N. Y.

Miniature Blower. Motor and blower in 1-in. cube moves 3 cu ft of air per min. Weight, 1 oz, power 4 watts. For aircraft and missiles. Sanders Associates, Inc., Nashua, N. H.

Subminiature Relay. Length 1⁵/₁₆ in., diam 1³/₄. Wt. 0.225 lb, 2 amps at 26.5 VDC or 115 VAC. 4 pdt. Will take 50 G. U.S. Relay, 1744 Albion St., Los Angeles, Calif.

Silicon Diode. Model 1N23D is designed for X-band mixer at low noise (7.5 db). Normal and reversed polarity models. Bomac Laboratories, Inc., Beverly, Mass.

Klystron Radar Transmitter. 50 kw peak, 1 kw average power for MTI applications. Power required is 40 amps per phase for 208 v 50/60 cps, 3-phase supply. Levinthal Electronic Products, 2910 Fair Oaks Ave., Redwood City, Calif.

Signal Simulator. 400 cycle suppressed carrier modulated signal generator for checking aircraft or missile autopilots. Micro Gee Products Inc., 6100 W. Slauson Ave., Culver City, Calif.

Sub-Miniature Rate Gyro. Weighing 4 oz, measures 2¹/₈ in. long and 1 in. diam. Rate range of 10-500 deg/sec. Frequencies of 8-75 cps. Robey Rotor Div., J. B. Rea Co., 1723 Cloverfield Blvd., Santa Monica, Calif.

High Temp Pressure Switch. Kilotrol switch allows operation to 800 F, burst pressure 10,000 psi; operating pressure of 3100 psi at 800 F. SPST ¹/₄ amp at 28 VDC contacts. Cook Electric Co., 2700 Southport Ave., Chicago 14, Ill.

Compact Power Rectifier. 30 W liquid-cooled germanium rectifier occupies volume of 190 cu. in. Current outputs of 540-750 maps. Input voltages of 26 v to 66 v rms. International Rectifier Corp., 1521 E. Grand Ave., El Segundo, Calif.

Materials

Metals

No Bead Tubing. Internal welding bead eliminated from tubing by new reverse process. Smooth and flawless interior. Crucible Steel Co. of America, Oliver Bldg., Pittsburgh 30, Pa.

Small Self-Locking Fasteners. Size ranges of 0-80 and 4-48. Standard Pressed Steel Co., Jenkintown, Pa.

Ni-SpanC. Nickel - chromium - iron-titanium alloy retains uniform elasticity at elevated temperatures. H. A. Wilson Co., Union, N. J.

Safety Well. Type 304 stainless steel protects thermocouple elements to 3000 psi and -300 F to +1850 F. Conax Corp., 7811 Sheridan Drive, Buffalo 21, N. Y.

JAN Shield Insert. Corrugated cadmium-plated brass insert conducts heat away from tubes. Inserts fit all 5¹/₂ and 6¹/₂ miniature envelopes. Birtcher Corp., 4371 Valley Blvd., Los Angeles 32, Calif.

Thermocouple Connector. Kit features interchangeability with various materials. Merlin Manufacturing Corp., 12410 Triskett Road, Cleveland 11, Ohio.

Plastics

Fuming Nitric Acid Filter. Woven Teflon housed in stainless steel. Pore sizes of 150 × 300 microns and 25 × 40 microns. Porous Plastic Filter Co., 30 Sea Cliff Ave., Glen Cove, N. Y.

Acid Resistant Hose. Fluorinated inner tube permits acid resistant operation from -100 to +500 F. Aeroquip Corp., 303 S. East St., Jackson, Mich.

Resistant Tubing. Fiberglas-epoxy tubing in lengths up to 9 ft and diameters down to ¹/₁₆ in. ID. Withstands temps to 350 F. Lamtex Industries, 51 State St., Westbury, L. I., N. Y.

Stripper for Epoxies and Polyesters. Solvent Monastrip EP for reclaiming pottings and castings. Mona Industries, Paterson 4, N. J.

Vinyl Chloride. Pliovic AO in organosols. Chemical Div., Goodyear Tire & Rubber Co., 1144 E. Market St., Akron 16, Ohio.

Plastic Patch Kit. Epoxy welds metals, nonmetals. Temperature and acid resistant. Amchem Corp., 823 Tuxedo, Highland Park 3, Mich.

Formica Tooling. Sheet S-52 for stretch forming, metal spinning and die forming. Formica Co., 4614 Spring Grove Ave., Cincinnati 32, Ohio.

Polyester Resin. Solid Atlac 382 resin for laminates, molding permits monomer to be added in desired amount. Atlas Powder Co., Wilmington 99, Del.

Miscellaneous

Silicone Polymers. New silicones are available with water solubility. For rubber, plastics, and lubricants. Linde Air Products Co., 30 E. 42nd St., New York 17, N. Y.

Thixotroping Agent. Cabo-O-Sil, a finely divided silicon dioxide permits lay-up with sharp corners, etc., in polyesterglass reinforced plastics. Godfrey L. Cabot, 77 Franklin St., Boston 10, Mass.

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- ROCKET TEST ENGINEERS

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Marquardt Aircraft Company
Van Nuys, California
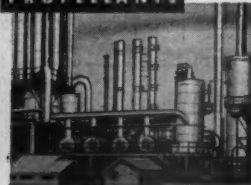
Olin Mathieson Chemical Corp.
New York, New York

Reaction Motors, Inc.
Denville, New Jersey

New Patents

George F. McLaughlin, Contributor

Electric ignition control in starting devices for turboengines (2,730,862). Rene L. Lamy, Paris, France, assignor to Société du Carburateur Zenith.

Means responsive to the speed of a turboengine during the starting period whereby a switch in the ignition circuit is closed as the speed reaches a value enabling the engine to run by its own power.
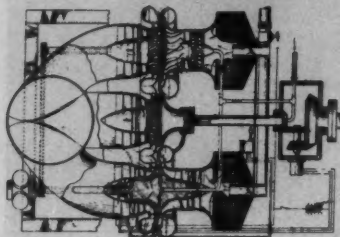
2,730,863

Gaseous fuel turbine power plant having parallel connected compressors (2,730,-863). Nathan C. Price, St. Helena, Calif., assignor to Lockheed Aircraft Corp.

Plurality of parallel units each including a compressor, a turbine and a load-driving turbine. A combustion chamber is in receiving relation to the outlets of the compressor, and delivers combustion gases to drive the load-driving turbines.

Circuit for measuring the average intensity of disturbances (2,730,896). Desolde R. de Boisblanc, Bartlesville, Okla., assignor to Phillips Petroleum Co.

System for determining combustion instability in a jet engine. An electromagnetic detector picks up acoustical noise from the burner and measures the output of an integrating circuit.
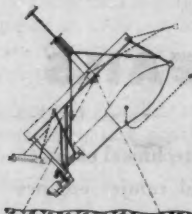
2,730,927

Launching device for self-propelled missiles (2,730,927). Louis Marie E. Bourguard, Courbevoie, France, assignor to Société d'Exploitation des Materials Hispano-Suiza.

Support for holding a rocket missile in the direction of firing. When the support is retracted simultaneously from all points of contact, the missile drops with its axis parallel to direction, and the propelling charge is immediately ignited.

Exhaust gas temperature responsive fuel system for gas turbine power plant (2,731,974). Bruce N. Torrell, Wethersfield, Conn., assignor to United Aircraft Corp.

Fuel flow metered as a function of exhaust gas temperature. System includes rotor driven by exhaust gases and system for maintaining a constant pressure ratio

across the rotor which actuates the fuel metering means.

Acoustic pulses jet engine with acoustic air intake system (2,731,795). Albert G. Bodine, Jr., Van Nuys, Calif.

Standing wave established within a resonant sonic column. An air-induction pipe has a length corresponding to one-quarter of the wave length of the wave pattern in the fluid column. The pipe resonates with the fluid column and the end of the pipe furthest from the combustion zone represents a zone of minimum pressure variation.

Gas turbine plant comprising a rotary regenerative heat exchanger (2,731,798). R. J. Welsh and R. D. van Millingen, Rugby, England, assignors to the English Electric Co., Ltd.

Process of operating a turbine plant with a fuel mixture of air and a methane concentration below 1 percent. Mechanical energy extracted from heated and compressed elastic fluid is used partly for compressing the fluid and partly for driving an external load.

2,731,885

Rocket stowage and launching mechanism (2,731,885). Auldin D. Nolan, Palos Verdes, Calif., assignor to Northrop Aircraft, Inc.

Cylindrical streamlined shell containing 19 rockets and mounted on a wing. Rockets stowed at the center of the shell are conveyed on pivotally mounted launching arms to external positions as the shell doors are swung open.

Control valve apparatus for turbojet engine (2,731,983). Leighton Lee II, Rocky Hill, Conn., assignor to Pratt & Whitney, Inc.

Spool valve with barometric control for increasing compressor pressure differential moves. A valve in the system is subject to tailpipe temperature. Lifting of a piston in the system permits increasing the torque.
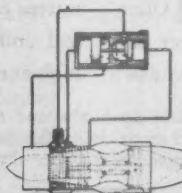
2,732,125

Differential area compressor bleed control (2,732,125). Lowell E. Ruby, Marlborough, Conn., assignor to United Aircraft Corp.

Multistage axial flow compressor in which the intermediate stage is bled to

move a piston as a function of compressor pressure ratio. Discharge pressure is admitted to the flow regulating means to control compressor bleeding.

Rocket sight for aircraft (2,733,006). Horace W. Babcock, Pasadena, Calif., assignor to the U. S. Navy.

Fire control sight with motor coupled sighting member and computer. A potentiometer with a sliding contact is mechanically coupled so that incremental movements of the sight are followed by corresponding movements of the contact.

Ejectable aircraft seat capsule (2,733,027). Doane R. Gero, Akron, Ohio, assignor to Goodyear Aircraft Corp.

One piece shell, completely open at front, on launching rails. A lower and upper door completely closing the shell are operated by gas power located within the capsule.
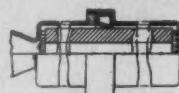
2,733,568

Solid propellant jet reaction motor (2,-773,568). Lionel A. Dickinson, Aylesbury, England, assignor to the British Government.

Propellant grain within a casing providing an annular space between its outer surface and the inside surface of the casing. The space communicates with the inside of the grain at both ends and an annular resilient sealing washer prevents the longitudinal flow of gas through the space in a rearward direction.
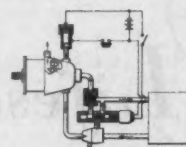
2,733,569

System for supplying liquid fuel to a combustion chamber (2,733,569). David R. Trowbridge, Hornchurch, England, assignor to the Plessey Co., Ltd.

Pressurized exhaust gases are initially supplied to the combustion chamber from a cartridge firing device. A regulating valve conducts the flow of low pressure fuel from the turbopump to the chamber.

Jet engine power regulator (2,734,340). Cyrus F. Wood, Swarthmore, Pa., assignor to Westinghouse Electric Co.

A normally balanced electrical network operated in response to engine speed and temperature is used for varying the fuel input and the area of a jet nozzle.

Gas turbine engine with means for reheating turbine exhaust gases (2,734,341). Alfred Cyril Lovesey, Derby, England, assignor to Rolls-Royce Ltd.

Closed end casing structure in the exhaust duct with a restriction maintaining the combustion chamber pressure approximately at that within the main combustion equipment.

Book Reviews

Ali Bulent Cambel, Northwestern University, Associate Editor

Mathematics of Engineering Systems, by Derek F. Lawden, M. A., Methuen, London; John Wiley, New York; 1954, viii + 380 pages. $5.75.

> Reviewed by JOHN J. MARTIN
> Bendix Products Division
> Bendix Aviation Corporation

"Mathematics of Engineering Systems" will serve as an excellent primer for the young engineer beginning in the dynamic analysis of physical systems, and will in addition be the ready reference, for the most part, for the practitioner of the art. The physical systems considered range through electronic amplifiers and oscillators, electric circuits, servomechanisms and regulators, and other mechanical and electrical systems. The book will be a blow to the feelings of some undergraduates who hold that their courses in mathematics and mechanics are needless complications to receiving a baccalaureate degree.

The book begins with a concise as well as complete introduction to fundamentals, including the definition of the function concept, limits and derivatives, integrals, and complex numbers. For the person without the mathematical background to take Chap. 1 in stride, further study would be indicated before the latter chapters are taken on. For the most part, Chap. 1 will serve as a sufficient review for the above average graduate engineer.

Chap. 2, without much ado, gets into the practice of solving linear differential equations with constant coefficients by classical methods, and applies these to the solution of practical problems. As the next obvious step, the chapter considers simultaneous differential equations of the type mentioned and brings this consideration to useful fruition in the study of the stability of linear systems in general, and of some common systems in particular.

Chap. 3 extends the work of Chap. 2 on linear differential equations with constant coefficients from the point of view of modern methods. For the most part, the introduction and use of the operational calculus are the substance of the chapter, and this is applied to linear systems. The chapter is concluded by the generalization of the Heaviside operational calculus to the method of the Laplace transform.

Chap. 4 deals with Fourier analysis and is a standard treatment beginning with the trigonometric series representation of periodic functions, continuing with a study of how the coefficients of this series are obtained, and giving some useful applications. These practical applications include those relating to the band width of an R.F. amplifier and to the servomechanism. After this, some of the advantages of the Fourier analysis are presented, and the chapter is concluded with a consideration of the Fourier integral in the infinite interval. Hence the text treats the operational calculus from Heaviside to the generalizations named for Laplace and Fourier.

In the final chapter, Chap. 5, some modern work applying to nonlinear differential equations is given and these range over the same problems treated earlier in the text. If the subject of the last chapter becomes of considerable interest to the reader, other texts on nonlinear mechanics may be of interest. Two good ones are "Non-Linear Mechanics" by Minorsky and "Introduction to Non-Linear Mechanics" by Kryloff and Bogoliuboff.

Running through the text are problems, the solutions for which are given in the back of the book. In adition, a short table of common Laplace transforms is included. In general, the book is highly recommended for those entering or engaged in systems analysis work.

A History of Mechanical Inventions, by Abbott Payson Usher, Harvard University Press, Rev. Ed., 1954, 450 pp. $9.

> Reviewed by R. S. HARTENBERG
> Northwestern University

Recent discoveries in physics, reduced to practice by advances in technology, have somewhat shaken the world. Just where and how these things will find their place in the future remains to be seen. The new horizons are only relative, however, for the past has had its technological and other impacts, too—a fascinating story unknown to many. An outline of this story is presented in this "History of Mechanical Inventions."

History, the study of past events, is, contrary to a generally held belief, neither static nor well defined, for it is the study of man's unrest. Knowledge derived from newly found archeological evidence, mislaid documents and the enlightened collation of old materials adds continuously to the story of how it was. This edition, a revision that covers a span of 25 years, reflects the winnowings of those years.

Abbott Payson Usher, Professor of Economics (Emeritus) at Harvard University, has an interest and understanding that goes beyond mechanical devices. The interplay of the forces of nature and their economic implications has been the author's concern in four other volumes: it is from this matrix of ideas that the present volume is justly derived. One may not agree with all points of Professor Usher's thesis, but one will find each thought-provoking.

Since mechanical inventions are many, and perhaps even impossible to list completely, a consideration of the chapter headings is necessary to show the scope of the over 400 pages of text. They are: I, The Place of Technology in Economic History; II, Historical Analysis of Social Change; III, The Particular System of Events; IV, The Emergence of Novelty in Thought and Action; V, The Early History of the Pure Applied Mechanical Sciences; VI, The Mechanical Equipment of Pre-Christian Antiquity; VII,

The Development of Water Wheels and Windmills: 150 B.C.–A.D. 1500; VIII, Water Clocks and Mechanical Clocks: 16 B.C.–A.D. 1500; IX, Leonardo da Vinci: Engineer and Inventor; X, The Invention of Printing; XI, Machinery of the Textile Industries: 100–1800; XII, The Development of Clocks and Watches into Instruments of Precision: 1500–1800; XIII, The Production and Application of Power: 1500–1830; XIV, Machine Tools and Quantity Production: 1450–1850; XV, The Production and Distribution of Power Since 1832.

The first four chapters are entirely new to this revision. Here a theory of invention and a concept of social evolution are presented. Gestalt psychology forms the basis of the argument; according to this, the mark of man's behavior is the manifestation or striving to achieve an end, where the solution to the problem is reached by an act of insight, a grasp of the logical significance of the whole situation, instead of by only the blundering tactics of trial and error.

The remainder of the book is in the form of a critical narrative, concerned in large part with the production of power. Necessarily the interdependence of industry and power, of mechanism and machine, are studied.

It is inevitable that a reader will find that he has questions that are left unanswered. In justice to this fine single volume he must realize that there was space for only selected topics, and that even these could not always be discussed in ultimate detail. The Notes and Bibliography are extensive; they show not only the tremendous wealth of source material but also the great diversification of sources.

This book is well worth reading, for it is not often that one finds the story of how the particular is imbedded in a total pattern, i.e., the relation of a part to the whole.

HALEY ELECTED CHAIRMAN OF ARS BOARD

At a meeting of the ARS Board of Directors in New York City on January 24, 1956, Mr. Andrew G. Haley, a Director of the Society, and Past President of the ARS, was elected Chairman of the Board of Directors, to serve in that capacity until the return to active duty of President Noah S. Davis, Jr. Mr. Davis is currently unable to serve as Chairman of the Board because of illness, although his return to this capacity is expected in the near future. This election was conducted in accordance with the provisions of Article VII, Section 4, of the ARS by-laws.

Mr. Haley was also named, at the last meeting, to membership on the ARS Space Flight Committee.

THE RV-A-10 MISSILE

powered by a "THIOKOL" solid propellant rocket motor

The RV-A-10 missile, developed jointly by the Army Ordnance Corps, General Electric, and Thiokol Chemical Corporation, is an example of technical progress in solid propellant rocketry. The rocket motor for this missile was designed and developed by the Thiokol Chemical Corporation. Its successful flight tests definitely established the feasibility of "Thiokol" solid propellant power plants for use in large, long range and high altitude missiles.

Development of the power plant for the RV-A-10 stems largely from Thiokol's "systems" concept of rocket motor development. Under this concept of rocket motor design, all of the requirements of the missile are taken into consideration before the propulsion unit is put on the drawing boards. The motor's thrust, duration, size and shape are all governed by the specific application for which the unit is intended.

Close coordination is conducted with the missile prime contractor to consider problems such as aerodynamic stability, center of gravity requirements, guidance system impact and vibration resistance, wing and fin attachments, and other over-all missile problems.

Engineers and Chemists — *become a member of Thiokol's rocket development and manufacturing team. We welcome inquiries from mechanical engineers, chemical engineers and chemists interested in the rocket field.*

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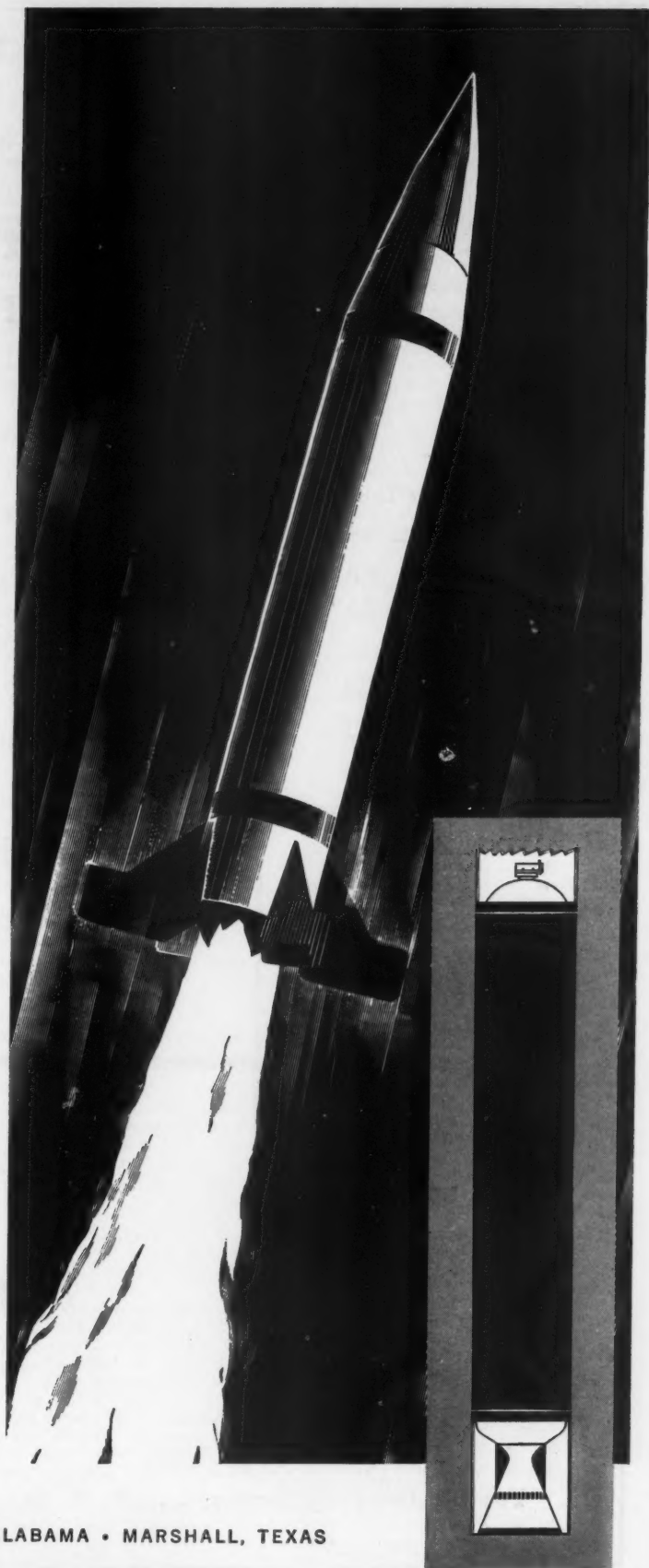
Technical Literature Digest

M. H. Smith, *Associate Editor*, and M. H. Fisher, *Contributor*
The James Forrestal Research Center, Princeton University

Jet Propulsion Engines

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